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Cov(X, Y) = b Var(X), Var(Y) = b2 Var(X) b Var(X) ρ (X ,Y) = 26. Letting 1 if U ≤ .6 , X1 = 0 otherwise X2 = 1 if U ≤ .4 , 0 otherwise X3 = 1 if U ≤ .3 0 otherwise Hence, it is not possible for all Xi to equal 1. (a) ∞ ∞ 10 − 10 = 1/ 2 . 2 2 Chapter 3 43 23. i 1 11 i =1 10 10 ∑ 46. Using that the sum of the dice is 9 with probability 1/9, we obtain upon conditioning on whether A rolls a 9 that PA = (b) 1/32, (c) 10/32, (d) 1/4, (e) 31/32. 89. (b) Use the Venn diagram below to obtain the answer 32/100. Then E[ounces of commodity] = 24. Suppose X is Weibull with parameters v, α, β. E[X|I] = μI , Var(X|I) = σ I2 Var(X) = E[σ I2] + Var(μ I) = pσ 12 + (1 − p)σ 22 + pμ12 + (1 − p) μ 22 − [pμ1 + (1 − p) μ 2]2 65. If A ⊂ B P(AB) P(AB) ≥ = P(AB|A ∪ B) P(A) P(A ∪ B) P(A) , P(A|Bc) = 0, P(B) P(A|B) = 3. a Γ(a + 1) Γ(a + b) = Γ(a + b + 1) Γ(a) a+b (a + 1)a Γ (a + 2) Γ (a + b) E[X2] = B(a + 2, b)/B(a, b) = = Γ (a + b + 2) Γ (a) (a + b + 1)(a + b) Thus, E[X] = B(a + 1, b)/B(A, b) = Var(X) = 26. (a) ∫ pdp = 1/ 2 0 1 (b) ∫ p dp = 1/ 3 2 0 1 71. Fi = Ei ∩ E cj 6. (b) P{MaxUi ≤ v} = P{U1 ≤ x, ..., Un ≤ x} = ∏P{Ui ≤ x} by independence = xn (c) Simulate n random numbers and use the maximum value obtained. ⎝n − i⎠58 Chapter 4 13. n 2(n − 1)(n − 2) 2 = in a line n! n 2 n (n − 2) ! 2 if in a circle, n ≥ 2 = n! n −1 (a) If A is first, then A can be in any one of 3 places and B's place is determined, and the others can be arranged in any of 3! ways. Hence, 70 percent smoke neither. n j =1 Let ⎧1 success on trial j Ij = ⎨ ⎩0 otherwise ⎡ E⎢ ⎣ n ∑ 1 ⎛ Var⎜⎜ ⎝ 13. = 10 = 3 7 185 (.6) + (.8) 10 10 7 8 9 12 13 14 (b) 3 (c) 5·6·7 12 · 13 · 14 (d) 3 26 7 ·8·5 12 · 13 · 14 5·6·7 12 · 13 · 14 Chapter 3 41. By assigning instruments to Jay, Jack, John and Jim, in that order, we see by the generalized basic principle that there are 2 · 1 · 2 · 1 = 4 possibilities. 55. (a) P{A} = P{next card is an ace}P{A|next card is an ace} 3 1 3 = = 32 4 128 (b) Let C be the event that the two of clubs appeared among the first 20 cards. 32. Let Z be a standard normal random variable. ⎛ 2N − k ⎞2N −k 2⎜ ⎟(1 / 2) N ⎝ ⎠⎛ 2 N − k − 1⎞2 N − k −1 2⎜ (1 / 2) ⎟(1 / 2) ⎝ N −1 ⎠Chapter 4 55 79. n Fx(j) (x) = ∑ ⎛⎜ ⎝ in ⎞⎟⎠F (x)[1 − F (x)] f X (j) (x) = ∑ ⎜⎝ i ⎟⎠F n −i i i= j n ⎛n⎞ i −1 (x) f (x)[1 − F (x)]n − i i= j − n ∑ ⎛⎜ ⎝ in ⎞⎟⎠F (x)(n − i)[1 − F (x)] n − i −1 i f (x) i= j n n! ∑ (n − i)!(i − 1)!F = i −1 (x) f (x)[1 − F (x)]n − i i= j − = 26. (a) 20/26 ≈ .769 (b) 20/(20 + 36) = 5/14 ≈ .357 (d) p ≈ P{Z ≥ (25.5 − 20)/ 20 } ≈ P{Z ≥ 1.23} ≈ .1093 (e) p = .112184 136 Chapter 8 Theoretical Exercises 1. EFc is the event that neither of the dice lands on 1 and the sum is odd. Then β = P(C|N) = P(NC) P(N) P (N C) P (C) = = = P(N C) P (C) + P (N C c) P(C c) α 1 2 α + (1 − α) 2α ≥α 1+α with strict inequality unless α = 1. (a. 86. (a) (10)!/3!4!2! ⎛ 3 ⎞7! (b) 3⎜ ⎟⎝ 2 ⎠4 !2 ! 30. 373, line -8: on the centered equation following "the preceding equation yields" add a right paren at the very end. (a) This follows because the conditional distribution of Y + Z given that Y = y is normal with mean y and variance 1, which is the same as the conditional distribution of X given that Y = y. 3 1 36 36 15 2 = = . To prove (b), use (a) and the boundary condition M0 = a (c) P{(n + 1)st is white} = E[P{(n + 1)st is white|Xn}] Mn ⎡ X ⎤ = E⎢ n ⎥ = ⎣a + b⎦ a + b 40. Then, if X is the number of the 1000 time periods in which the stock increases, then its price at the end is ⎛u⎞ suXd1000-X = sd1000 ⎜ ⎟⎝d⎠X Hence, in order for the price to be at least 1.3s, we would need that X ⎛u⎞d1000 ⎜ ⎟> 1.3 ⎝d⎠or X> log(1.3) − 1000 log(d) = 469.2 log(u / d) That is, the stock would have to rise in at least 470 time periods. k ⎤ n ∑ X ∑ X ⎥⎦ = k / n i i 1 1 Let ⎧1 outcome j never occurs Ij = ⎨ ⎩0 otherwise r r Then X = ∑ ∫ (1 − p) I j and E[X] = j 1 12. Method (a) gives too much weight to cars carrying a lot of workers. It is easy to see that P{X1 = i, X2 = j} = P{X1 = j, X2 = i} and so X1 and X2 have the same distribution. The ordering will be unchanged if for some k, 0 ≤ k ≤ n, the first k coin tosses land heads and the last n − k land tails. Let B and W be the events that the marble is black and white, respectively, and let B be the event that box i is chosen. Then ⎡ ⎤ E⎢ X j ⎥ = ⎢⎣ j ≠1 ⎥⎦ ∑ = ∑ E[X j] j ≠1 ∑ P{type j before type 1} j ≠1 = ∑ P /(P + P) , j j 1 j ≠1 where the last equality follows upon conditioning on the first time either a type 1 or type j is caught to give. Let C be the event that the tumor is cancerous, and let N be the event that the doctor does not call. Letting E be the event that the eldest son, letting S be the event that it is a son, and letting A be the event that the child's family has at least one son, P(ES) P(S) = 2P(E) 3 1⎤⎡ = 2⎢ P(E A) + P(E Ac) ⎢4 4⎥⎣ 1⎤⎡1 3 + 0 ⎥ = 3/4 = 2⎢ 4⎥⎣2 4 P(E|S) = Chapter 3 35 (b) Using the preceding notation P(ES) P(S) = 2P(E) 7 1⎤⎡ = 2⎢ P(E A) + P(E Ac) ⎢8 8⎥⎣⎡1 7 ⎤ = 2⎢ ⎥ = 7/12 ⎣3 8 ⎦P(E|S) = 76. (a) (b) (c) (d) 8. ⎛12 ⎞ 8 ⎞(20)! (12) 20⎢ ⎥ ⎢4 4 4 4 ⎝ ⎠ ⎠(3!) (2!) 49. [X] + 1 has a geometric distribution with parameter p = 1 − e −λ and x − [X] is distributed as an exponential with rate λ conditioned to be less than 1. Since all i! orderings among these cards are equally likely it follows that⎡ n ⎤ E[Ji] = 1/i! and thus E[N] = E⎢ J i ⎥ = ⎣ i =1 ⎦∑ 18. 16 Chapter 2 54. 4 3 1 P(B) (b) P(B|A) = = 52 51 = 48 47 P(A) 1 − 33 52 51 12. i i 1 ⎛n − Proposition 6.2 gives the result⎥ ⎥ ⎝ 21. n ∑X i =1 ⎥⎥ 25 so need n = 10 / n − 75 > 5⎬ ≤ ⎥⎦ 25n Let Z be a standard normal random variable. E1, E2, ..., En are conditionally independent given F if for all subsets i1, ..., ir of 1, 2, ..., n ( ) ∏ P(E P Ei1 ...Eir F = 44 r j =1 ij F). ⎛2⎞(a)⎢ ⎢ ⎝3⎠5 5 3 6 2 7 8 ⎛ 8 ⎞⎛ 2 ⎞⎛ 1 ⎞⎛ 8 ⎞⎛ 2 ⎞⎛ 1 ⎞⎛ 8 ⎞⎛ 2 ⎞⎛ 1 ⎞⎛ 2 ⎞(b)⎢ ⎢ ⎥⎢ ⎥+⎢ ⎢ ⎥⎢ ⎥+⎢ ⎢ ⎥⎢ ⎥+⎢ ⎥⎜ 5 ⎟⎜ 3 ⎟⎜ 3 ⎟⎜ 6 ⎟⎜ 3 ⎟⎜ 3 ⎟⎜ 7 ⎟⎜ 3 ⎟⎜ 3 ⎟⎜ 3 ⎟5 ⎛ 5 ⎞ 2 ⎞ 1 (c)⎢ ⎢ ⎥⎜ 4 ⎟⎜ 3 ⎟3 ⎛ 6 ⎞ 2 ⎞(d)⎢ ⎢ ⎥⎜ 4 ⎟⎜ 3 ⎟5 ⎛1⎞⎥ ⎥⎜3⎟2 76. i i =1 ⎡N ⎤ E[number of occupied tables] = E⎢ X i ⎥ = ⎣ i =1 ⎦Now, ∑ N ∑ E[X] i i =1 E[Xi] = P{ith arrival is not friends with any of first i − 1} = (1 − p)i−1 and so N E[number of occupied tables] = ∑ (1 − p) i −1 i =1 Chapter 7 99 7. None are true. E[score] = p*[1 − (1 − P)2 + (1 − p*)(1 − p2) d = 2(1 − p)p* − 2p(1 − p*) dp = 0 ⇒ p = p* 32. ⎝2 ⎠(f) Since the events Bi are mutually exclusive P(∪ Bi) = ∑ P(B) = (2 i n ⎛ 2n ⎞− 1) ⎥ ⎥= (1 / 2) n −1 ⎝2 ⎠81. e − y2 / 2 ∑ E[X i, j] i< j dy = e− x 2 ⎛n⎞1 1 = ⎥ ⎥= ⎝ 2 ⎠n(n − 1) 2 /2 2π Let Ii equal 1 if guess i is correct and 0 otherwise. 1 {E[X 2 Y = 1] + E[X 2 Y = 2] + E[X 2 Y = 3]} 3 1 = {9 + E[(5 + X) 2] + E[(7 + X) 2]} 3 1 = {83 + 24 E[X] + 2 E[X 2]} 3 1 = {443 + 2 E[X 2]} since E[X] = 15 3 E[X 2] = Hence, Var(X) = 443 − (15)2 = 218. Hence, by Jensen's Inequality E[Y n/(n−1)] ≥ E[Y])n/(n−1) Now set Y = X n−1 and so E[X n] ≥ (E[X n−1])n/(n−1) or (E[X n])1/n ≥ (E[X n−1])1/(n−1) 21. ⎛n⎞∑⎜ ⎝ i ⎟⎠x ∑ ∑ n = i1 1 (x2 i1 = 0 1 i1 1 i 2 ,..., ir ⎛ n − i1 ⎞i2 i2 ⎢ ⎥x1 ...xr ⎝ i2 ,..., ir ⎠i 2 + ...+ ir = n − i1 = ∑∑...∑ i1 ,..., i r ⎛ n ⎞i1 ir⎢ ⎥x1 ...xr ⎝ i1 ,..., ir ⎠i1 + i2 + ...+ i r = n where the second equality follows from the induction hypothesis and the last from the ⎛ n ⎞⎛ n − i1 ⎞⎛ n ⎞identity ⎢ ⎥ ⎥=⎢ ⎥. Then 9 of the 18 workers were in a car carrying 9 workers and so if you randomly choose a worker then with probability 1/2 the worker would have been in a car carrying 9 workers and with probability 1/2 the worker would have been in a car carrying 1 worker. e−3s/60 + (s/20)e−3s/60 8. P(E1 ∪ E2 ∪ E3 ∪ E4) = P(E1) + P(E2) + P(E3) + P(E4) − P(E1E2) − P(E1E3) − P(E1E4) − P(E2E3) − P(E2E4) + P(E3E4) + P(E1E2E3) + P(E1E2E4) + P(E1E3E4) + P(E2E3E4) − P(E1E2E3E4) = P1P4 + P1P3P5 + P2P5 + P2P3P4 − P1P3P4P5 − P1P2P4P5 − P1P2P3P4 − P1P2P3P5 − P2P3P4P5 − 2P1P2P3P4P5 + 3P1P2P3P4P5. 5! (.45)2(.15)(.40)2 2! 1!2! 12. E[X] = 60 =6 10 Var(X) = Chapter 7 20(80) 3 7 336 = from Example 5c. 56. P{X = i|X + Y = n} = P{X = i, Y = n − i}/P{X + Y = n} = 15. (a) 1 11 (1 + 2 + ... ∑ ∞ ∞ ∑∑ P{N = k} P{N ≥ i} = i =1 i =1 k =1 ∞ ∞ = ∑∑ P{N = K} k =1 i =1 ∞ = ∑ kP{N = k} = E[N] . Now generate a random number U and set X equal to X1 if U < p and equal to X2 if U > p. No. f(5/2) < 0 4. If λ(t) = ctn then the distribution function is given by 1 − F(t) = exp{−ktn+1}, t ≥ 0 where k = c/(n + 1) Hence, using the inverse transform method we can generate a random number U and then set X such that exp{−kXn+1} = 1 − U or X = {−log(1 − U)/k}1/(n+1) Again U can be used for 1 − U. fY(y) = e−y 1 ∫ ye −x / y dx = e − y . P{Braves win} = P{B|B wins 3 of 3} 1/8 + P{B|B wins 2 of 3} 3/8 + P{B|B wins 1 of 3} 3/8 + P{B|B wins 0 of 3} 1/8 1 3⎡ 1 1 3 ⎤3 3 1 38 = +⎥ + + = 8 8⎣ 4 2 4 ⎦⎦ 8 4 2 64 where P{B|B wins i of 3} is obtained by conditioning on the outcome of the other series. (a) P1P2(1 − P3)(1 − P4) + P1(1 − P2)P3(1 − P4) + P1(1 − P2(1 − P3)P4 + P2P3(1 − P1)(1 − P4) + (1 − P1)P2(1 − P3)P4 + (1 − P1)(1 − P2)P3P4 + P1P2P3(1 − P4) + P1P2(1 − P3)P4 + P1(1 − P2)P3P4 + (1 − P1)P2P3P4 + P1P2P3P4. 65. C − Ap = 28. Let A be the event that this person attends the party and let W be the event that this person is a woman. ∫ ∫ E[g(X)/f(X)] = [g (x) / f (x)] f (x)dx = g (x)dx Chapter 10 143 Corrections to Ross, A FIRST COURSE IN PROBABILITY, seventh ed. There are 23. Then ⎧1 n ⎥⎥ 2 2n − 1 , E[Xi] = ⎨ 1⎡ (n − 1)(n − 2) ⎤ ⎥ ⎥1 − ⎦, ⎥ ⎥2 ⎣ (2n − 1)(2n − 2) ⎦ if i = 1, 2n otherwise Therefore, ⎡ n ⎤ E⎢ Xi ⎥ = ⎣ i =1 ⎦∑ 2n ∑ E[X] i i =1 = 1 ⎛ 2n 3n ⎞+ (2 n − 2)⎥ ⎢2 ⎝2n − 1 4n − 2 ⎠= 3n 2 − n 4n − 2 (b) In the case of a round table there are no end positions and so the same argument as in part (a) gives the result ⎡ (n − 1)(n − 2) ⎤3n 2 n ⎢1 − ⎥= ⎣ (2n − 1)(2n − 2) ⎦4n − 2 where the right side equality assumes that n > 1. 1 − (9 / 11)15 1 − (9 / 11)30 82. (a) P{cX ≤ a} = P{X ≤ a/c} and differentiation yields 1 λ f X (a / c) = e − λ a / c (λ a / c)t −1 Γ(t) . p(0, 0) = (12/13)3(11/12)3 p(0, 1) = p(1, 0) = (12/13)3[1 − (11/12)3] p(1, 1) = (2/13)[(1/13) + (12.13)(1/13)] + (11/13)(2/13)(1/13) Chapter 6 77 y 8. u uv ,x= v +1 v +1 (a) u = x + y, v = x/y = y, v = x/y ⇒ y = u , 0 < uv < 1 + v, 0 < u < 1 + v (v + 1) 2 y1 = x1 + x2, y2 = e x1 . E[X] = ∫ 1 y 2π y>x 17. Because⎡ r ⎤ E⎢ Xi ⎥ = ⎣ i =1 ⎦∑ r ∑ E [X i] = i =1 r ∑ 2(1/ 2) Ai =1 it follows that for at least one coloring the number of monocolored subsets is less than or equal to 17. (a) P(A) = P(A|C)P(C) + P(A|C c)P(C c) > P(B|C)P(C) + P(B|C c)P(C c) = P(B) (b) For the events given in the hint P(C A) P (A) (1/ 6)(1/ 6) = = 1/ 3 P(A|C) = 3/ 36 3/ 36 Because 1/6 = P(A is a weighted average of P(A|C) and P(A|Cc), it follows from the result P(A|C) > P(A) that P(A|C c) < P(A). Let M be the event that the person is male, and let C be the event that he or she is color blind. 1 − (1 − p)n ≥ 1/2, or, n ≥ − Chapter 3 log(2) log(1 − p) 41 i −1 12. Let W and F be the events that component 1 works and that the system functions. Hence, P(E b F) = 77. (a) P(S ∪ H ∪ D ∪ C) = P(S) + ... − P(SHDC) 3 4 ⎛ 2⎞⎛ 2 ⎞⎛ 48 ⎞⎛ 2 ⎞⎛ 46 ⎞⎛ 2 ⎞⎛ 44 ⎞4⎢ ⎢6⎢ ⎢ ⎢ ⎢4⎢ ⎥⎢ ⎥⎢ ⎥⎢ ⎥⎢2 7 2 5 2 2 2 9 = ⎝ ⎠− ⎝ ⎠⎝ ⎠+ ⎝ ⎠⎝ ⎠− ⎝ ⎠⎝ ⎠52 52 52 52 ⎛ ⎞⎛ ⎞⎛ ⎞⎛ ⎞ ⎥⎢ ⎥⎢ ⎥⎢13 13 13 ⎝ ⎠⎝ ⎠⎝ ⎠⎝ 13 ⎠ = ⎛ 50 ⎞⎛ 48 ⎞⎛ 46 ⎞⎛ 44 ⎞4⎢ ⎢−6⎢ ⎢+ 4⎢ ⎢−⎢ ⎢11 9 ⎝ ⎠⎝ ⎠⎝7⎠⎝5⎠⎛ 52 ⎞⎢ ⎢⎝ 13 ⎠⎛ 48 ⎞⎛13 ⎞⎛ 44 ⎞⎛13 ⎞⎛ 40 ⎞13⎢ ⎥⎢ ⎥⎢ ⎥⎢ ⎥⎢9 2 5 3 1 (b) P(1 ∪ 2 ∪ ... ∪ 13) = ⎝ ⎠− ⎝ ⎠⎝ ⎠+ ⎝ ⎠⎝ ⎠ ⎛ 52 ⎞⎛ 52 ⎞⎛ 52 ⎞⎥ ⎥⎥ ⎢13 13 ⎝ ⎠⎝ ⎠⎝ 13 ⎠56. j ⎠⎝ k ⎛ j =0⎥ 9 Chapter 2 Problems 1. The joint moment generating function, E[etX+sY] can be obtained either by using ∫∫ e E[etX+sY] = tX + sY f (x, y)dy dx or by noting that Y is exponential with rate 1 and, given Y, X is normal with mean Y and variance 1. 16 + x A direct check now shows that 9 sophomore girls (which the above shows is necessary) is also sufficient for independence of sex and class. ⎛ 26 ⎞P(En) = ⎢ ⎥⎝ 36 ⎠6 , 36 ∞ 2 ∑ P(E) = 5 n n =1 Imagine that all 10 balls are withdrawn P(A) = 28. k =1 ∞ 7. (a) P{H, T, T|6 heads} = P(H, T, T and 6 heads}/P{6 heads} = P{H, T, T}P{6 heads|H, T, T}/P{6 heads} ⎛7⎞⎛10 ⎞= pq 2⎢ ⎥p 5 q 2⎢ ⎥p 6 q 4 5 ⎝ ⎠⎝6⎠=1/10 (b) P{T, H, T|6 heads} = P(T, H, T and 6 heads}/P{6 heads} = P{T, H, T}P{6 heads|T, H, T}/P{6 heads} ⎛7⎞⎛10 ⎞= q 2 p⎢ ⎥p 5 q 2⎢ ⎥p 6 q 4 5 ⎝ ⎠⎝6⎠=1/10 51. Now suppose that n + m = k + 1. Thus, 1 − e−n/365 ≥ 1/2 when n/365 ≥ log(2). (a) 1 − e−1/2 1 1 − e −1 / 2 − e −1 / 2 2 (b) P{X ≥ 2|X ≥ 1} = 1 − e −1 / 2 (c) 1 − e−1/2 (d) 1 − exp {−500 − i)/1000} 66. Now 0 0 1 ∫ x− y a 1 x ∫ ∫ dy = (x − y) dy + (y − x) a dy 0 a 0 x x 1− x ∫ = u a du + 0 ∫ u du a 0 = [x a +1 + (1 − x) a +1] /(a + 1) Hence, 1 a E[X − Y] = = 98 1 [x a +1 + (1 − x) a +1]dx a +1 0 ∫ 2 (a + 1)(a + 2) Chapter 7 4. Hence, P{Xi = 1} = .6 and so ∑ P{X = 3} = P{X1 = X2 = X3 = 1} ≤ P{Xi = 1} = .6. be independent" p. 288, l. 27 51 27 51 · 27 (.02)(.5) 10 = (.02)(.5) + (.03)(.3) + (.05)(.2) 29 42. 1 ≥ P(E ∪ F) = P(E) + P(F) − P(EF) 12. ∞ (b) E[N(x)] = ∑ P{N (x) > n = n=0 63. ∫ P{max > t}dt (a) E[max] = 0 1 ∫ = (1 − P{max ≤ t })dt 0 1 n n +1 ∫ = (1 − t n / dt = 0 1 (b) E[min] = ∫ p{min > t}4t 0 1 ∫ = (1 − t) n dt = 0 27. Suppose that r labelled subsets of respective sizes n1, n2, ..., nr are to be made up from r n −1 ⎛ ⎞ni . 60. Parts (a), (b), (c), and (d) are immediate. The number of people in a random collection of size n that have the same birthday as yourself is approximately Poisson distributed with mean n/365. [∑ X = x]+ ... ⎛ n −1 ⎞⎛ n −1 ⎞By Proposition 6.1, there are⎢ ⎥=⎢ ⎥such solutions. ,X n be independent" to "Let X 1, . F(α) = 1 − e−1 24.⎡ 100 ⎤1 E⎢ Li ⎥ = 1000 + 50(101) = 1505 10⎣ i =1 ⎦2 ⎛ 100 ⎞100 ⎛ i ⎞1 100 2⎢ ⎥i Var⎢ Li ⎥ = ⎢10 + ⎥= (100) 2 + (100)(101) + 10 ⎠100 i =1⎝ i =1 ⎠i =1⎝ ∑ ∑ ∑ ∑ Now apply the central limit theorem to approximate. 73. u = x + y, v = x + z, w = y + z ⇒ z = v+ w−u v−w+u w−v+u ,x= ,y= 2 2 2 1 1 0 J = 1 0 1 = −2 0 1 1 f(u, v, w) = 59. Take logs and apply the central limit theorem 8. (a) By the generalized basic principle of counting there are 26 · 26 · 10 · 10 · 10 · 10 · 10 = 67,600,000 (b) 26 · 25 · 10 · 9 · 8 · 7 · 6 = 19,656,000 2. By Proposition 6.1 there are ∑ ∑ ⎛ n − 1⎞⎛ m − 1⎞⎛ n − 1⎞⎛ m − 1⎞⎢ ⎢ ⎥outcomes of the first type and⎥ ⎥of the second. ∑ Must choose c so that 1 ∫ .01 = 5(1 − x) 4 dx = (1 − c)5 c so c = 1 − (.01)1/.5. (b) Let xi,j equal 1 if there is an edge between vertices i and j, and let it be 0 otherwise. ⎛ j − 1⎞P{Y = j} = ⎢ ⎥⎝ n − 1⎠N ⎛ j − 1⎞E[Y] = ⎢ ⎥n − 1⎠j =n ⎝ ∑ = = = = 31. Similarly, for u ≥ 1/4, F−1(u) is the solution of x − x2/12 − 2 = u that falls in the region 3 ≤ x ≤ 6. P{S found} To compute these probabilities, condition on the cabinet selected. a. 23. Now given X(n) = s, X(1) , ..., X(n−1) are distributed as the order statistics of a set of n − 1 uniform (0, s) random variables. (b) The probability that an arbitrary couple were born on the same day of the year is 1/365. (a) 48 8! ⎛ 8 ⎞= 2520. 5 ⎛ 39 ⎞265 (b) Same method as used in (a) yields the answer 5⎥ + 1 ⎥= . (a) P{correct|agree} = P{correct, agree}/P{agree} = p2/[p2 + (1 − p)2] = 36/52 = 9/13 when p = .6 (b) 1/2 66. The final equation used that P(A|a, b) = P(A|b, a) = P(A). (a) P{J3 votes guilty|J1 and J2 vote guilty} = P{J1, J2, J3 all vote guilty}/P{J1 and J2 vote guilty} 7 3 (.7)3 + (.2)3 97 10 = 10 . Then, using the binomial theorem, E[X n] = n ⎛n⎞∑⎜ i ⎟⎠σ E[Z]μ i i n −i i =0 Now make use of theoretical exercise 46. Condition on outcome of initial trial P(E before F) = P(E b F) = P(E b F|E)P(E) + P(E b F|F)P(F) + P(E b F|neither E or F)[1 − P(E) − P(F)] = P(E) + P(E b F)(1 − P(E) − P(F)]. Let X denote the number of blue genes that Smith received. (a) 1 − P(A ∪ B) = 1 − (.07 + .28 − .05) = .7. ⎛n ⎞⎛n ⎞P⎢ ∪ Ei ⎥ = 1 − P⎢ ∩ Eic ⎥ = 1 − ⎝1 ⎠⎝1 ⎠7. Suppose that each element is, independently, equally likely to be colored red or blue. Hence, if someone buys 3 packages then the probability they will return exactly 1 is 3p(1 − p)2. n Note that X = ∑ i =1 Xi + m ∑Y . If we let Xi denote the time between the ith and (i + 1)st failure, i = 0, ..., n − 2, then it follows from Exercise 9 that the Xi are independent exponentials with rate 2λ. P{ X > 30} 1 / 4 = 1/3 = P{ X > 10} 3 / 4⎡ 50 ⎤(a) exp⎢ − λ (t) dt ⎥ = e−.35⎢⎣ 40 ⎦⎦∫ (b) e−1.21 36. (a) [I − (1 − P1P2)(1 − P3P4)]P5 = (P1P2 + P3P4 − P1P2P3P4)P5 (b) Let E1 = {1 and 4 close}, E2 = {1, 3, 5 all close} E3 = {2, 5 close}, E4 = {2, 3, 4 close}. ∞ 1 2 −x / 2 xe dx = 2 y 2e − y dx = 2Γ(3) = 4 (a) E[X] = 40 0 ∫ ∫ (b) By symmetry of f(x) about x= 0, E[X] = 0 ∞ (c) E[X] = 5 ∫ x dx = ∞ 5 1 7. P{cX < x} = P{X < x/c} = 1 − e−λx/c 15. (a) e−2.5 (b) 1 − e−2.5 − 2.5e−2.5 − Chapter 4 (2.5) 2 − 2.5 (2.5)3 − 2.5 e − e 2 3! 53 64. P(D) = .36, P(C) = .30, P(C|D) = .22 (a) P(DC) = P(D) P(C|D) = .0792 (b) P(D|C) = P(DC)/P(C) = .0792/.3 = .264 18. 58. X is Poisson with mean λ = 2 and Y is Binomial with parameters 10, 3/4. 90 Chapter 6 8. ⎝ 2⎠⎝ 2⎠⎝ 2⎠(b) There are 6 · 7 choices of a math and a science book, 6 · 4 choices of a math and an economics book, and 7 · 4 choices of a science and an economics book. (a) No (b) Yes, since fY(x|I = 1) = fX(x) = fX(−x) = fY(x|I = 0) (c) fY(x) = 1 1 f X (x) + f X (− x) = f X (x) 2 2 (d) E[XY] = E[E[XY|X]] = E[XE[Y|X]] = 0 (e) No, since X and Y are not jointly normal. 1 This works by the law of large numbers since E[k(U)] = ∫ k (x)dx . For instance P{B|B win 2 of 3} = P{B|D or G win 3 of 3, B win 2 of 3} 1/4 = P{B|D or G win 2 of 3, B win 2 of 3} 3/4 11 3 + . 11 1 3 + 22 2 11 1 (b) P{fair|hh} = 4 2 = . (b) (e) ⎛ 5⎞6⎢ ⎢5 · 4 · 3 ⎝ 2 ⎠65 ⎛ 5⎞6 · 5⎢ ⎥⎝ 3 ⎠65 (c) (f) ⎛ 6⎞⎛ 5⎞ 3⎞⎢ ⎢4⎢ ⎥⎥⎝ 2⎠⎝ 2 ⎠⎝ 2 ⎠65 ⎛5⎞6 · 5⎢ ⎥⎝ 4 ⎠65 6 65 ∏i 8 2 i =1 64 · 63 · · · 58 18. 44. Let A denote the event that this customer carries an American Express card and V the event that he or she carries a VISA card. on line 12: Pn , m −1, m → Pn , m −1 • • p. n 10. Let Ci denote the event that relay i is closed, and let F be the event that current flows from A to B. Thus, this probability is smaller when the correlation is .2. r − 1 ⎠n=r ⎝ ∑ 16. Therefore P(Bi), the probability that none of the other n − k contestants beats all the members of a given subset of k contestants, is [1 − (1/2)k]n−k. 2π Hence, R and θ are independent with θ being uniformly distributed on (0, 2π) and R having density fR(r) = 2r, 0 < r < 1. Therefore, 74 percent of the establishment's customers carry at least one of the two types of credit cards that it accepts. 2 !2 ! 2!1! Thus, by the basic principle, there are 18 different paths from A to B that go through the circled piont. With S being survival and C being C section of a randomly chosen delivery, we have that .98 = P(S) = P(S|C).15 + P(S|C2) .85 = .96(.15) + P(S|C2) .85 Hence 17. = 10 = 7 3 26 2(.7)(.3) + 2(.2)(.8) 10 10 (c) P{J3 guilty| J1, J2 vote innocent} 7 3 (.7)(.3) 2 + (.2)(.8) 2 33 10 . 2 x ⎤ − x ⎡∞ − x ⎤ 5! ⎡ −x f X (3) (x) = ⎢ xe dx ⎢xe⎢ xe dx ⎢2!2! ⎣⎦ 0⎢ ⎣ x ⎦⎥ ⎦2 −2x −x −x = 30(x + 1) e xe [1 − e (x + 1)]2 ∫ ⎛ L − 2d ⎞⎢ ⎢⎝ L ⎠3/ 4 ∫ 3 3/ 4 48. P{in black} = P{5 black}α P{5 black}α + P{5 white}(1 − α) 1 2 2π = 1 2 2π e −(5− 4) α = α 2 2 e −1 / 8 + 2 e −(5− 4) 2 α + (1 − α) /8 α /8 1 3 2π e − (5− 6) 2 / 18 e −1 / 8 (1 − α) −1 / 8 e 3 α is the value that makes preceding equal 1/2 Chapter 5 69 A 31. 12. • p. b ∫ (a + bx)dx = 1 or a + 3 = 1 2 0 1 3 ∫ x(a + bx)dx = 5 2 or 0 a= 3 6 , b= 5 5 ∞ 8. From Example 3g, 1 + 6 6 6 6 + + +6 5 4 3 2 23. Gamma (n, λ) 52. Using the hint P{A ⊂ B} = n ⎛ n⎞(2i / 2n)⎢ ⎢2n = ⎝i⎠i =0 ∑ n ⎛ n⎞∑⎜ i ⎟2 / 4 i n = (3/4)n i =0 where the final equality uses n ⎛n⎞∑⎜ i ⎟2 1 i n −i = (2 + 1)n i =0 38 Chapter 3 (b) P(AB = φ) = P(A ⊂ Bc) = (3/4)n, by part (a), since Bc is also equally likely to be any of the subsets. P{D >α} = P{⎢X − μ⎢ > αμ} ≤ 3. Hence, ∑ ∑ P {∑ X i } > 3 ≈ P{⎢N(0, 1)⎢ > 3(12/50)1/2} by the central limit theorem = 2P{N(0, 1) > 1.47 = .1416 6. (a) Using the fact that f integrates to 1 we see that 1 c(n, i) ≡ ∫x i −1 (1 − x) n − i dx = (i − 1)!(n − i)!/n!. = 1/ 8 1/ 2 + 1 1/ 2 (b) 1/18 71. For part (e), we have the following: ∑ ⎛n⎞⎢ ⎢i i =0 ⎝ ⎠∑ k !n ! n! ⎛n⎞k⎢ ⎥= = k (n − k)!k ! (n − k)!(k − 1)! ⎝⎠(n − k + 1)n! n! ⎛ n ⎞= (n − k + 1)⎢ ⎥= k − 1 ⎝ ⎠(n − k + 1)!(k − 1)! (n − k)!(k − 1)! n(n − 1)! n! ⎛ n − 1⎞= n⎢ ⎥= (n − k)!(k − 1)! (n − k)!(k − 1)! ⎝ k − 1⎠Chapter 1 5 11. (a) 20 · 18 · 16 · 14 · 12 · 10 · 8 · 6 20 · 19 · 18 · 17 · 16 · 15 · 14 · 13 ⎛10 ⎞ 9 ⎞8! 6⎢ ⎢2⎝ 1 ⎠⎝ 6 ⎠2! (b) 20 · 19 · 18 · 17 · 16 · 15 · 14 · 13 53. 138 Chapter 8 Chapter 9 Problems and Theoretical Exercises 1. (b) The first equality in the hint follows since the permutation given will be the permutation after insertion of element k if the previous permutation is i1, ..., ij−1, i, ij, ..., ik−2 and the random choice of one of the k positions of this permutation results in the choice of position j. Y2 ≥ Y + 2 ⇔ Y ≥ 2 and so P{Y2 ≥ Y + 2} = P{Y ≥ 2} = 3/5. ∑λ 2n n /(2n)! = e−λ(eλ + e−λ)/2 log P{X = k} = −λ + k log λ − log (k!) ∂ k log P{ X = k} = −1 + λ ∂λ =0⇒λ=k Chapter 4 59 19. The number of subsets of size k that have i as their highest numbered member is equal to ⎛ i −1 ⎞⎢ ⎥, the number of ways of choosing k − 1 of the numbers 1, ..., i − 1. (a) 2p(1 − p) ⎛3⎞(b)⎢ ⎥p 2 (1 − p) ⎝ 2 ⎠(c) P{up on first|up 1 after 3} = P{up first, up 1 after 3}/[3p2(1 − p)] = p2p(1 − p)/[3p2(1 − p)] = 2/3. E[Xn] = 1 ∫ x dx = n + 1 n 0 n P{X ≤ x} = P{X ≤ x1/n} = x1/n 1 ⎛1 ⎞1 1⎢ −1 ⎥1 1/ n 1 x dx = E[X] = x x ⎝ n ⎠dx = n n0 n +1 0 n 15. 2. (a) P{Y > X} = P{Y − X > 0} ≈ P{Y − X > .5} ⎧⎢ Y − X − (160 − 170) .5 − (160 − 170) ⎥⎥ = P⎨ > ⎬ (20) 2 + (15) 2 (20) 2 + (15) 2 ⎥⎦ ⎢⎝ ≈ P{Z > .42} ≈ .3372 (b) P{X + Y > 350} = P{X + Y > 350.5} ⎧⎢ X + Y − 330 = P⎨ > ⎢⎝ (20) 2 + (15) 2 ≈ P{Z > .82} ≈ .2061 Chapter 6 ⎥⎥ ⎬ (20) 2 + (15) 2 ⎥⎦ 20.5 83 34. i =1 (a) E[R] = E[E[R|S]] = E[S] = μ (b) Var(R|S) = 1, E[R|S] = S Var(R) = 1 + Var(S) = 1 + σ2 ∫ f (s)F = C∫e μ (c) fR(r) = S R S (r 2 − (s −) / 2σ s)ds 2 e−(r − s) 2 /2 ds ⎧⎢ ⎛ μ + rσ 2 ⎞⎥ = K exp⎨− ⎢⎝ S − 1 + σ 2 ⎥⎠⎥⎝⎛ ∫ ⎛ σ 2 ⎞⎥ ⎥ds exp {−(ar2 + br)} 2⎥⎥ 2 ⎥⎬ 1 σ + ⎥⎠⎥ ⎝ Hence, R is normal. P(Y > X) = ∑ P(Y > X X = j) p j j = ∑ P(Y > j X = j) p j j = ∑ P(Y > j) p j j = ∑ (1 − p) j p j j 36. P(A ∪ V) = P(A) + P(V) − P(AV) = .24 + .61 − .11 = .74. Then, P(A) = P(A|B0)P(B0) + P(A|B1)P(B1) + P(A|B2)P(B2) + P(A|B3)P(B3) ⎛ 6 + i ⎞⎛ 6 ⎞⎛ 9 ⎞ ⎥⎥ ⎢ ⎢3⎢ ⎝ 3 ⎠⎝ i ⎠⎝ 3 − i ⎠= 15 ⎞⎛15 ⎞i =0⎛ ⎢ ⎥⎥ ⎝3⎠⎝3⎠= .083 ∑ 30. with 3: P{pass} = n n ⎛n⎞i i=k ⎝ ⎠⎛n⎞i i=k ⎝ ⎠1⎡ ⎛ 3 ⎞2 2⎡ ⎛ 3 ⎞2 3⎤3⎤⎥ 2 ⎢(.8) (.2) + (.8) ⎥+⎥ 2 ⎢(.4) (.6) + (.4) ⎢3⎣⎝ ⎠⎦3⎣⎝ ⎠⎦ = .533 with 5: P{pass} = 1 5 ⎛ 5⎞i 2 5 ⎛ 5⎞i 5−i 5−i⎢ ⎢(.8) (.2) + ⎢ ⎢(.4) (.6) 3 i =3⎝ i ⎠3 i =3⎝ i ⎠∑ ∑ = .3038 9 47. However, a loss would either be $1 or $3. Hence, there are 94 possible choices. Then Pij = P1Pi−1,j + (1 − P1) Pij Pij = P2 Pi , j −1 + (1 − P2) Pij . Let F = E1. In the n case, consider P{X(k) − X(k−1) > t|X(n) = s}. 100 Chapter 7 11. 1 − lim F (a − h) 4. (a) 13131 9 = 2 4 2 4 2 128 2. ⎛ 4⎞⎢ ⎢2 6 P{X = 4} = ⎝ ⎠= 14 ⎛ ⎞91 ⎢ ⎥⎝2⎠⎝ 2⎠⎢ ⎢2 1 P{X = 0} = ⎝ ⎠= 14 ⎛ ⎞91 ⎢ ⎥⎝2⎠⎛ 4 ⎞ 2 ⎞⎢ ⎢2 1 8 P{X = 2} = ⎝ ⎠⎠= ⎛14 ⎞91 ⎢ ⎥⎝2⎠⎛ 8 ⎞ 2 ⎞⎢ ⎢1 1 16 P{X = −1} = ⎝ ⎠⎠= 91 ⎛14 ⎞⎢ ⎥⎝2⎠⎛ 4 ⎞ 8 ⎞⎢ ⎢1 1 32 P{X = 1} = ⎝ ⎠⎠= 91 ⎛14 ⎞⎢ ⎥2⎝ ⎠⎛8⎞⎢ ⎥2 28 P{X = −2} = ⎝ ⎠= ⎛14 ⎞91 ⎢ ⎥⎝2⎠p(1) = 1/36 p(5) = 2/36 p(9) = 1/36 p(15) = 2/36 p(24) = 2/36 p(2) = 2/36 p(6) = 4/36 p(10) = 2/36 p(16) = 1/36 p(25) = 1/36 p(3) = 2/36 p(7) = 0 p(11) = 0 p(18) = 2/36 p(30) = 2/36 p(4) = 3/36 p(8) = 2/36 p(12) = 4/36 p(20) = 2/36 p(36) = 1/36 5 5 5 5 45 5 = = , P{X = 3} = , 10 9 18 10 9 8 36 5 4 3 5 10 5· 4·3· 2 5 5 = , P{X = 5} = = P{X = 4} = , 10 9 8 7 168 10 · 9 · 8 · 7 6 252 5 · 4 · 3 · 2 ·1 1 = P{X = 6} = 10 · 9 · 8 · 7 · 6 252 4. But from Table 5.1, P{Z < 1.28} = .90 and so (c − 12)/2 = 1.28 or c = 14.56 20. i i k k =1 ⎝ ⎠i =0 ⎝ ⎠i =0 ⎝ ⎠∑ ∑ ∑ 11. (a) 5! = 120 7! (b) = 1260 2!2! 11! (c) = 34,650 4!4!2! 7! (d) = 1260 2!2! 9. By exercise 28 ⎛ j −1 ⎞⎛ m + n⎞P{Y = j} = ⎢ ⎥⎢ ⎣m≤j≤n+m ⎝ m − 1⎠⎝ m ⎠Now, X = n + m − Y and so ⎛ m + n − i − 1⎞⎛ m + n ⎞P{X = i} = P{Y = m + n − i} = ⎢ ⎥⎢ ⎣ i ≤ n ⎝ m −1 ⎠⎝ m ⎠32. Also, let p denote the proportion of the population that is male. P{Boy, F} = P(WF) P(W) 1/ 2 = = c P(F) 1 − P(F) 1 − (1 / 2) n −1 4 16 + x P{Boy) = so independence = 4 = 10 16 + x P{F} = 10 16 + x 10 · 10 ⇒ 4x = 36 or x= 9. (a) The inverse transform method shows that U1/n works. A = {1,0001,0000001, ...} B = {01, 00001, 00000001, ...} (A ∪ B)c = {00000 ..., 001, 000001, ...} 5. Let Fn denote the fortune after n gambles. 130 Chapter 7 46. As there are⎢ ⎢groups of size r that consist of i ⎝ r ⎠⎝ i ⎠⎝ r − i ⎠men and r − i women, we see that ⎛ n + m⎞⎢ ⎥= ⎝ r ⎠n r ⎛ n ⎞⎛ m ⎞∑⎜ i ⎟⎜ r − i ⎟ . ⎛ 4 ⎞⎛ 52 ⎞(a)⎢ ⎥⎢ ⎥≈ .0045, ⎝ 2⎠⎝ 2 ⎠⎛ 4 ⎞⎛ 52 ⎞(b) 13⎢ ⎥⎢ ⎥= 1/17 ≈ .0588 ⎝ 2 ⎠⎝ 2 ⎠37. Now E[X1] = 3j and so E[2 + X1 + X2] = 48 by the results of Example 5 106 . + (r + b)(r + b + g) r + b + g r + g (b) P(RBG) = 8. 50 Chapter 4 37. 86 Chapter 6 1− a a + x P(X(4) − X(1) ≤ a} = ∫ ∫ 48xy(y 2 − x 2)dydx 1− a 0 51. (a) P(Ind|voted) = P(voted Ind) P(Ind) ∑ P(voted type) P(type) = .35(.46) ≈ 331 .35(.46) + .62(.3) + .58(.24) (b) P{Lib|voted} = .62(.30) ≈ .383 .35(.46) + .62(.3) + .58(.24) (c) P{Con|voted} = .58(.24) ≈ .286 .35(.46) + .62(.3) + .58(.24) (d) P{voted} = .35(.46) + .62(.3) + .58(.24) = .4862 That is, 48.62 percent of the voters voted. 27. 11. Chapter 7 117 67. 437 123 = 35 35 E[(X − Y)]2 = Var(X − Y) = Var(X) + Var(−Y) = 2σ2 ⎛ 10 ⎞Var⎢ X i ⎥= 10 Var(X 1) . We can now generate X from F by generating a random number U and setting X = F−1(U). 1 − μy μe (b) Interpret Xi as the number of trials needed after the (i − 1)st success until the ith success occurs, i = 1, ..., n, when each trial is independent and results in a success with probability p. ⎛5⎞3 ⎛5⎞4 ⎛3⎞2 2 5 3⎢ ⎥p (1 − p) + ⎢ ⎥p (1 − p) + p ≥⎢ ⎥p (1 − p) + p ⎝ 4⎠⎝2⎠⎝3⎠⇔ 6p3 − 15p2 + 12p − 3 ≥ 0 ⇔ 6(p − 1/2)(p − 1)2 ≥ 0 ⇔ p ≥ 1/2 43. Then P(∪i4=1 Ai) = 4 2 · 7! 2 2 · 6! 23 · 5! 24 · 4! −6 +4 − 8! 8! 8! 8! and the desired probability is 1 minus the preceding. ∑ r i =1 (1 / 2) Ai −1 Var(λX 1 + (1 − λ) X 2) = λ2σ 12 + (1 − λ) 2 σ 22 d (dλ) = 2λσ 12 − 2(1 − λ)σ 22 = 0 ⇒ λ = [σ 22 σ 12 + σ 22] As Var(λX1 + (1 − λ)X2) = E (λX 1 + (1 − λ) X 2 − μ) 2 we want this value to be small. (b) 0 25. 10 ∑ 1 ⎡ 1 1 ⎛ 1 ⎞2 ⎤180 18 ⎞1 15 + 10 · 9⎢ −⎥ ⎥= X i ⎥= 10 2 19 19 ⎠⎦19 17 ⎛ 19 ⎞⎦ (19) 17 E[R] = n(n + m + 1)/2 ⎤⎡ n+m 2 ⎢ i 2 ⎢nm⎢ i =1 ⎣ n + m +1 ⎥ ⎥Var(R) = −⎢ ⎢n + m −1 ⎢ n + m ⎝ 2 ⎠⎦ ⎥ ⎦ The above follows from Example 3d since when F = G, all orderings are equally likely and the problem reduces to randomly sampling n of the n + m values 1, 2, ..., n + m. Let X denote Jill's score and let Y be Jack's score. From this we see that 0 E[X(i)] = c(n + 1, i + 1)/c(n, i) = i/(n + 1) i (i + 1) E[X (2i)] = c(n + 2, i + 2)/c(n, i) = (n + 2)(n + 1) 126 Chapter 7 and thus i(n + 1 − i) (n + 1) 2 (n + 2) Var(X(i)) = (b) The maximum of i(n + 1 − i) is obtained when i = (n + 1)/2 either 1 or n. But, P(ABC) = 1/4. If A chooses (b) then B chooses (a). 5 7 7 9 35 − . On the other hand, if E[f(X)] ≥ E[f(Y)] for all increasing functions f, then by letting f be the increasing function f(x) = 1 if x > t 0 otherwise then P{X > t} = E[f(X)] ≥ E[f(Y)] = P{Y > t} and so X >st Y. In each case B wins probability 5/9. P{second child brown} = P{brown|Smith blue}1/2 + P{brown|Smith no blue}1/2 = 1/4 + 1/2 = 3/4 Chapter 3 31 61. E[1/(X + 1)] = i =0 n = ∑ i =0 = 1 n! ∑ i + 1 (n − i)!i! p (1 − p) i n −i n! p i (1 − p) n −i (n + 1)!(i + 1)! n ⎛ n + 1⎞ p ∑ ⎜ i + 1 ⎟p i +1 (1 − p) n − i ⎡ (n + 1) 1 0 1 n+1− 0 ⎤1 − ⎢ 0 ⎢p (1 − p) ⎥(n + 1) p⎣ ⎦⎦ = [1 − (1 − p) n + 1] (n + 1) p = 11. Let R be the event that she receives a job offer. Also, for j < k k −1 E[XjXk] = ∏ n ∏ (1 − 2 / i) (1 − 1 / i) i= j i =k Hence, for j < k, k −1 ∏ Cov(Xj, Xk) = i= j n Var(X) = ∑ E[X n ∏ (1 − 1/ i) i=k j](1 − (1 − 2 / i) − n ∏ i= j n (1 − 1/ i) (1 − 1 / i) i=k E[X j]) + 2Cov(X j , X k) j =1 33. It is the distribution of the sum of t independent exponentials each having rate λ. 48 Chapter 4 (b) E[N] = 3[p3 + (1 − p)3 + 4[3p2(1 − p)p + 3p(1 − p)2(1 − p)] + 5[6p2(1 − p)2 = 6p4 − 12p3 + 3p2 + 3p + 3 Differentiation yields d E[N] = 24p3 − 36p2 + 6p + 3 dp Its value at p = 1/2 is easily seen to be 0. (d) N(3) = 1 + 3N(1) + 3N(2) = 1 + 3 + 9 = 13 N(4) = 1 + 4N(1) + 6N(2) + 4N(3) = 75 17. i =0 ⎛ n ⎞⎛ n⎞⎢ ⎢= i n − i⎠i =0 ⎝ ⎠n 2 9. (a) .8 (b) .3 (c) 0 9. By (d) and the induction hypothesis we have that Pn,m = n n −1 − m m n − m +1 n − m + = n + m n −1+ m n + m n + m n + m − 1 n + m which completes the proof. 1/2 10. If Xi is the outcome of the ith roll then E[Xi] = 7/2 Var(Xi] = 35/12 and so ⎛ 79 ⎤ ⎛ 79 ⎞ P⎨ X i ≤ 300⎬ = P{ X i ≤ 300.5⎬ ⎝ i =1 ⎭ ⎝ i =1 ⎭ ⎧ 300.5 − 79 (7 / 2) ⎫ ≈ P⎨ N (0,1) ≤ ⎬ = P{N (0,1) ≤ 1.58} ≈ .9429 (79 × 35 / 12)1 / 2 ⎭ ⎝ ∑ Chapter 8 ∑ 133 7. 79, line 2: P (∑ j =1 R j) → P(U nj =1 R j) • p. Then E[X1 + X2] = 80 Var(X1 + X2) = Var(X1) + Var(X2) + 2 Cov(X1, X2) = 72 + 2[.6(6)(6)] = 93.6 (a) With Z being a standard normal 90 − 80 ⎞(P(X1 + X2 > 90) = P⎢ Z > ⎥93.6 ⎠⎝ = P(Z > 1.034) ≈ .150 120 Chapter 7 (b) Because the mean of the normal X1 + X2 is less than 90 the probability that it exceeds 90 is increased as the variance of X1 + X2 increases. (b) 1 − e−.2 − .2e−.2 = 1 − 1.2e−.2 (a) e−.2 Since each letter has a small probability of being a typo, the number of errors should approximately have a Poisson distribution. 1/ 2 P{S found A}1/2 + P{S found B}1/2 = 1 2 = . Now, i =1 j =1 m i m j =1 j =1 j =1 i +1 ∑ i − j = ∑ (i − j) + ∑ (j − i) = [i(i − 1) + (m − i)(m − i + 1)]/2 m Hence, using the identity ∑j 2 = m(m + 1)(2m + 1)/6, we obtain that j =1 ⎡ 1 m(m + 1)(2m + 1) m(m + 1) ⎤ (m + 1)(m − 1) E[X − Y] = ⎢ 2 − ⎥= 6 2 3m ⎣m ⎦5. That is, (1 - (1 - pi) n → (1 - (1 - pi) n)) • • • • n p. Differentiation yields that d E[N] = 2 − 4 p dp and so the minimum occurs when 2 − 4p = 0 or p = 1/2. (a) If Xi has distribution Fi , i = 1, ..., n, then, assuming independence, F is the distribution of MaxXi. Hence, we can simulate from F by simulating Xi, i = 1, ..., n and setting X = MaxXi. (b) Use the method of (a) replacing Max by Min throughout. (a) 1/16 (b) 1/16 (c) The only way in which the pattern H, H, H, H can occur first is for the first 4 flips to all be heads, for once a tail appears it follows that a tail will precede the first run of 4 heads (and so T, H, H, H will appear first). (a) P{X = j, Y = i} = 11 , j = 1, ..., j, i = 1, ..., j 5 j (b) P{X = j|Y = i} = 1 5j 5 ∑ k =i 1/ 5 k = 1 j 5 ∑1/ k , 5 ≥ j ≥ i. + 10) = 10 2 (b) after 2 questions, there are 3 remaining possibilities with probability 3/5 and 2 with probability 2/5. The events {X > n} and {Y < r} are both equivalent to the event that there are fewer than r successes in the first n trials; hence, they are the same event. E[Fn] = E[E[Fn|Fn−1]] = E[2(2p − 1)Fn−1p + Fn−1 − (2p − 1)Fn−1] = (1 + (2p − 1)2)E[Fn − 1] = [1 + (2p − 1)2]2E[Fn−2] # = [1 + (2p − 1)2]nE[F0] 68. 2 Hence by Proposition X 2n is a gamma random variable with parameters (n, 1/2) and the result now follows from part (a). P(E1 ... En) ≥ P(E1 ... En−1) + P(En) − 1 by Bonferonni's Ineq. ( ) With c = 1/ 2π σ we have f(x) = ce − (x − μ) 2 / 2σ 2 f ′(x) = − ce− (x − μ) 2 / 2σ 2 2 (x − μ) /σ 2 2 2 f ′′(x) = cσ −4e− (x − μ) 2 / 2σ (x − μ) 2 − cσ −2e− (x − μ) / 2σ Therefore, f ′′(μ + σ) = f′′(μ − σ) = cσ−2e−1/2 − cσ−2e−1/2 = 0 ∞ 11. ⎝ i1 ⎠⎝ i2 ,..., in ⎠⎝ i1 ,..., ir ⎠20. n+m (d) Pn,m = P{A always ahead} = P{A always|A receives last vote} n n+m + P{A always|B receives last vote} = m n+m m n Pn−1,m + Pn , m −1 n+m n+m (e) The conjecture of (c) is true when n + m = 1 (n = 1, m = 0). ⎛ n ⎞ j ⎞⎛n ⎞ n − i ⎞(a)⎢ ⎥= ⎢ ⎥ ⎢ ⎢j ⎝ i ⎠⎝ j − i ⎠n (b) From (a), ⎛ n ⎞ j ⎞⎛n ⎞⎛n − i ⎞⎛n ⎞⎛n − i ⎞n −i j =i j =i (c) n∑ (−1) j =0 ⎝ k ⎠∑ 6 Chapter 1 15. 3/ 4 (b) Since we have no information about the ball in the urn, the answer is 1/2. b. 28. Hence, the probability of structural damage is { P{Wn − A ≥ 0} ≈ P Z ≥ (400 − 3n) / .09n + 1600 } Since P{Z ≥ 1.28} = .1 the probability of damage will exceed .1 when n is such that 400 − 3n ≤ 1.28 .09n + 1600 The above will be satisfied whenever n ≥ 117. Let Xii be the number of coupons one needs to collect to obtain a type i. P(E) + P(F) (a) This is equal to the conditional probability that the first trial results in outcome 1 (F1) given that it results in either 1 or 2, giving the result 1/2. 11/50 1 3 3 1 + P(BAs) 52 21 52 51 1 (a) P(B|As) = = = 2 P(As) 17 52 Which could have been seen by noting that, given the ace of spades is chosen, the other card is equally likely to be any of the remaining 51 cards, of which 3 are aces. P{Y = i, X = i} 1 = P{ X = i} 36 P{ X = i} 2 For j < i: P{Y = j|X = i} = 36 P{ X = i} For j = i: P{Y = i|X = i} = Hence i 1= ∑ P{Y = j X = i} = ⎧ 1 ⎥ P{Y = j⎥X = i} = ⎨ 2i − i ⎥ 2 ⎥ 2i − 1 and 36 and so, P{X = i} = ⎩ 2i − 1 42. (b) 1/2 (c) Intuitively, they should be independent. ⎧1 roll i lands on 1 Let Xi = ⎨ , ⎩0 otherwise ⎧1 roll i lands on 2 Yi = ⎨ ⎩0 otherwise Cov(Xi, Yj) = E[Xi Yj] − E[Xi]E[Yj] ⎧ 1 i = j (since X iY j = 0 when i = j ⎥− = ⎨ 36 ⎥1 − 1 =0 i≠ j ⎥ 36 36 Cov ∑ X , ∑ Y = ∑∑ Cov(X ,Y) i i j j i i =− 37. E[X] = ≥ ∑ i P{ X = i} = i +1 k ∞ ∑ i P{ X = i} = k +1 ∑ i P{X = k} i =1 = P{X = k}k(k + 1)/2 k2 ≥ P{ X = k} 2 7. 21. If n in the room, P{all different} = 12 · 11 · 12 · 12 · · (13 − n) · 12 When n = 5 this falls below 1/2. With C denoting this event, and Oi the event that the ith offspring is albino, we have: P(O1) = P(O1|C)P(C) + P(O1|Cc)P(Cc) = (1/4)(2/3) + 0(1/3) = 1/6 (b) P(O2 O1c) = = P(O1cO2) P(O1c) P(O1cO2 C) P(C) + P(O1cO2 C c) P(C c) 5/6 (3 / 4)(1 / 4)(2 / 3) + 0(1 / 3) 3 = = 5/6 20 62. P{X = 1} = 11/66 ⎛ 12 − j ⎞ 11 ⎢ 66 ⎠⎝ 54 + j = 2 11 P{X = 2} = ∑ ⎥⎝ P{X = 3} = ∑∑⎥⎝ k ≠1 k≠ j P{X = 4} = 1 ⎞ ⎠j ⎥⎦ ⎞11 ⎛ 12 − j ⎞ 12 − k ⎞ ⎥⎥ ⎥⎥ 66 ⎠⎝ 54 + j ⎠⎝ 42 + k ⎠⎠j =2 3 ∑ P{X = 1} i =1 16. F(y) = 0 for y < 10. If the first trial is a success, then the remaining n − 1 must result in an odd number of successes, whereas if it is a failure, then the remaining n − 1 must result in an even number of successes. For (a) and (c), see theoretical Exercise 18 of Chapter 6. Also, let Yi, i = 1, ..., m equal 1 if the ith small pill created is still in the bottle after the last large pill has been chosen and its smaller half returned. As it is equally likely to any of the n + m balls the result follows. Let X equal the number of lefthanders. Hence, with probability 2/3 the non-albino child is a carrier. 6 j ∑ 2 p(0, j) j =1 1 (42 + 10.5) = 52.5/12 12 (a) 6 · 6 · 9 = 324 (b) X = (6 − S)(6 − W)(9 − R) (c) E[X] = 6(6)(6)P{S = 0, W = 0, R = 3} + 6(3)(9)P{S = 0, W = 3, R = 0} + 3(6)(9)P{S = 3, W = 0, R = 0} + 6(5)(7)P{S = 0, W = 1, R = 2} + 5(6)(7)P{S = 1, W = 0, R = 2} + 6(4)(8)P{S = 0, W = 2, R = 1} + 4(6)(8)P{S = 2, W = 0, R = 1} + 5(4)(9)P{S = 1, W = 2, R = 0} + 4(5)(9)P{S = 2, W = 1, R = 0} + 5(5)(8)P{S = 1, W = 1, R = 1} ⎤1⎡ ⎛9⎞⎛ 6⎞⎛9⎞⎛6⎞⎛6⎞216⎢ ⎥+ 324⎢ ⎥+ 420 · 6⎢ ⎥+ 384⎢ ⎥9 + 360⎢ ⎥6 + 200(6)(6)(9)⎥⎥ ⎛ 21⎞⎝3⎠⎝3⎠⎝ 2⎠⎝ 2⎠⎝ 2⎠⎦ ⎝3⎠≈ 198.8 = 3. We must show that for any ordering of the 47 components there is a block of 12 consecutive components that contain at least 3 failures. ⎧⎥ P{X1 = r1, ..., Xn = rn} = P ⎨ X 1 = r1 ,..., X n = rn ⎥⎨ n ∑ ri ⎛ × e−λ λ 1 ⎥ ⎥ n ∑ r events⎬ i `1 ⎟ n ∑ r ⎥⎭ i =1 n ⎛ n ⎞ri ri ⎬ ∑ −λ 1⎢ 1 rn e λ r1 ⎠⎟P1 ... ⎛ 2n ⎞⎥ = ⎝n⎠10. When evaluated at 0, this gives that E[Z2j] - (2j)!/(j!2j) 47. Then ⎩0 − − fX X 1 ,..., X n + m (x x1 ,..., xn + m) = P{x1 ,..., xn + m X = x} P{x1 ,..., xn + m } = cx∑ i (1 − x) x n+ m− f X (x) ∑ xi n +m and so given ∑X = n the conditional density is still beta with parameters n + 1, m + 1. φY(t) = E[etY] = E[et(aX+b)] = etbE[etaX] = etbφX(ta) 49. P{D win} = P{G win} = 34 13 . (b) Note that F(x) = 1 2 F1 (x) + F2 (x) 3 3 where F1(x) = 1 − e−3x, 142 x > 0, F2(x) = x, 0 < x < 1 Chapter 10 Hence, using (a) let U1, U2, U3 be random numbers and set X= − log(U1) / 3 if U3 < 1 / 3 U2 if U3 > 1 / 3 where the above uses that −log(U1)/3 is exponential with rate 3. (b) Because cases 1 and 2 are mutually exclusive Pn = Pn−1 + (1 − Pn−k−1)(1 − P)pk 25. (a) Condition on the coin flip P{throw n is red} = 14 12 1 + = 26 26 2 3 3 1⎛ 2⎞1⎞1⎥ ⎥+⎥ ⎥P{rrr} 2⎝ 3 ⎠2⎝ 3 ⎠3 = (b) P{r|rr} = = 2 2 P{rr} 3 ⎛ 2 ⎞1 ⎛ 1 ⎞5 ⎢+⎥ ⎥⎢2⎝ 3 ⎠2⎝ 3⎠⎝ 22 ⎠1 ⎥ ⎥3 ⎢2 P{rr A}P(A) ⎝ ⎠(c) P{A|rr} = = = 4/5 2 2 P{rr} 2 ⎛ ⎞1⎛ ⎞1 ⎥ ⎥+⎢ ⎥⎝ 3⎠⎝ 2 ⎠3 2 4 8 7 6 4 8 7 6 4 8 7 6 543 + + + 12 12 11 10 9 12 11 10 9 8 7 6 12 11 10 9 8 7 4 8 7 6 54 8 7 6 54324 + P(B wins) = 12 11 12 11 10 9 8 12 11 10 9 8 7 6 5 8 7 4 8 7 6 544 8 7 6 54321 P(C wins) = + + 12 11 10 12 11 10 9 8 7 12 11 10 9 8 7 12 11 10 9 8 7 6 5 84. (a) Cov(X, Y|Z) = E[XY − E[X|Z]Y − XE[Y|Z] + E[X|Z]E[Y|Z] |Z] = E[XY|Z] − E[X|Z] E[Y|Z] − E[X|Z]E[Y|Z] + E[X|Z]E[Y|Z] = E[XY|Z] − E[X|Z]E[Y|Z] where the next to last equality uses the fact that given Z, E[X|Z] and E[Y|Z] can be treated as constants. Also, let Wi equal 1 if exactly one of A and B choose item i and let it be 0 otherwise. The number of partitions that has n + 1 and a fixed set of i of the elements 1, 2, ..., n as a ⎛n⎞subset is Tn−i. Not true. Hence, the probability of at least one such pair is 1 − e−219.18 ≈ 1. (a) p(0, 0) = (10/13)(9/12) = 15/26 p(0, 1) = p(1, 0) = (10/13)(3/12) = 5/26 p(1, 1) = (3/13)(2/12) = 1/26 (b) p(0, 0, 0) = (10/13)(9/12)(8/11) = 60/143 p(0, 0, 1) = p(0, 1, 0) = p(1, 0, 0) = (10/13)(9/12)(3/11) = 45/286 p(i, j, k) = (3/13)(2/12)(10/11) = 5/143 if i + j + k = 2 p(1, 1, 1) = (3/13)(2/12)(1/11) = 1/286 4. n (c) ⎛n⎞n ⎛ n ∑⎜ i ⎟⎠N (n − 1) = ∑ ⎢ n − i ⎟⎠N (n − i) i =1 i =1 n −1 = ⎛n⎞∑ ⎜ j ⎟⎠N (j) j =0 where the final equality followed by letting j = n − i. 39. (a + 1)a a2 ab − = 2 (a + b + 1)(a + b) (a + b) (a + b) (a + b + 1)(a + b) 2 (X − a)/(b − a) Chapter 5 75 28. (a) .6. Hence we may conclude that X and Y are independent and uniformly distributed on (−3/2, 3/2) Therefore, 3/ 2 E[|X| + |Y|] = 2 6. Then, ⎛⎢ P⎨ ⎢⎝ 4. But from the normal σ ⎠ ⎝ σ table P{Z < .84} ≈ .80 and so .84 ≈ 4/σ or σ ≈ 4.76 That is, the variance is approximately (4.76)2 = 22.66. (a) fX|Y(x|y) = (b) fY|X(y|x) = xe− x (y +1) ∫ xe− x (y +1) dx xe− x (y +1) ∫ xe − x (y +1) ∞a/x P{XY < a} = 2(i − 1) 1 + 36 P{ X = i} 36 P{ X = i} ∫ ∫ xe dy j=i j X2 + X3} + P{X2 > X1 + X3} + P{X3 > X1 + X2} = 3 P{ X1 > X2 + X3 } =3 ∫∫∫ dx dx dx 1 2 3 (take a = 0, b = 1) x1 > x2 > x3 0 ≤ xi ≤1 i = 1, 2 ,3 1 1− x3 1− x1 − x3 =3 ∫ ∫ ∫ dx dx dx 1 0 0 1 1 − x3 1 2 3 =3 ∫ ∫ (1 − x 2 0 − x3)dx2 dx3 0 1 =3 (1 − x3) 2 dx3 = 1 / 2 . 79. As E[Xi|Y] = Y, this gives that E[X1X2] = E[E[X1X2|Y]] = Ei[Y2], E[Xi] = E[E[Xi|Y]] = E[Y] Consequently, Cov(X1, X2) = E[X1X2] − E[X1]E[X2] = Var(Y) 34. ⎝2⎠(d) There will be 2n − 1 losers, and thus that number of games. 12 14 16 18 768 Chapter 3 21 15. P{Y − X > L/3} = 4 dydx L2 y− x> L / 3 ∫ ∫ L 5000} = P⎨Z > ⎬ 325.27 ⎠ ⎭ = P{Z > 1.8446} ≈ .0326 (b) P{X > 2000} = P{Z > (2000 − 2200)/230} ≈ P{Z > −.87} = P{Z < .87} = .8078 Hence, the probability that weekly sales exceeds 2000 in at least 2 of the next 3 weeks p3 + 3p2(1 − p) where p = .8078. (a) 0 2 λ 2 E[X] = 2 / λ2 b+a 2 (b) μ (c) 1 − e−λm = 1/2 or m = 13. Suppose P{X = b} = ε > 0 and bn = b + 1/n. Use the Chernoff bound: e−tiM(t) = eλ (e to satisfy t −1) − ti will obtain its minimal value when t is chosen λet = i, and this value of t is negative provided i < λ. Now if each successful trial is counted with probability p, than the number counted is Binomial with parameters n (large) and αp (small) which is approximately Poisson with parameter αpn = λp. 2 (a) n (b) Conditioning on whether the man of couple j sits next to the woman of couple j gives the 1 1 n−2 2 2n − 3 result: + = n − 1 n − 1 n − 1 n − 1 (n − 1) 2 (c) e−2 68. ⎛ j + n − 1⎞There are⎢ ⎥nonnegative integer solutions of j ⎝ n ∑x i = j i =1 Hence, there are Chapter 1 ∑ j + n − 1⎞ ⎥such vectors. We are using the results of Exercise 11. m !n ! (n + m − 1)! For 0 < x < 1 P{[X] = n, X − [X] < x} = P(n < X < n + x) = e−nλ − e−(n + x)λ = e−nλ(1 − e−xλ) Because the joint distribution factors, they are independent. 6 5 9 8 15 14 13 12 6. P{6 ⎪ different} = P{6, different}/P{different} P{1st = 6,2nd ≠ 6} + P{1st ≠ 6,2nd = 6} = 5/6 2 1/ 6 5 / 6 = 1/3 5.6 could also have been solved by using reduced sample space—for given that outcomes differ it is the same as asking for the probability that 6 is chosen when 2 of the numbers 1, 2, 3, 4, 5, 6 are randomly chosen. ⎛ 7 ⎞⎛ 8 ⎞ ⎢ ⎢3! 3 3 (a) ⎝ ⎠⎝ ⎠= 1/18 ⎛ 8 ⎞⎛ 9 ⎞ ⎥ ⎥4! ⎛ 4 ⎞⎛ 4 ⎞⎛ 7 ⎞⎛ 8 ⎞ ⎥ ⎥3 3 (b) ⎝ ⎠⎝ ⎠−1/18 = 1/6 ⎛ 8 ⎞⎛ 9 ⎞ ⎥ ⎥⎛ 4 ⎞⎛ 4 ⎞⎛ 7 ⎞⎛ 8 ⎞⎛ 7 ⎞⎛ 8 ⎞ ⎥ ⎥⎥ ⎥⎥ ⎥3 4 4 3 (c) ⎝ ⎠⎝ ⎠⎝ ⎠= 1/2 ⎛ 8 ⎞⎛ 9 ⎞ ⎥ ⎥⎛ 4 ⎞⎛ 4 ⎞31. 3· Chapter 4 A 1⎞ ⇒ C = A⎢ p + ⎥10 10 ⎠4 = 3/5 20 49 29. The number of carp is a hypergeometric random variable. ⎛ 4 ⎞⎛ 48 ⎞⎛ 52 ⎞⎛ 3 ⎞⎛ 36 ⎞⎛ 39 ⎞P(E2|E1) = ⎢ ⎢ ⎢P(E1) = ⎢ ⎢ ⎢, 1 12 13 ⎝ 1 ⎠⎝ 12 ⎠⎝ 13 ⎠⎝ 1 ⎠⎝ 12 ⎠⎝ 13 ⎠⎛ 2 ⎞⎛ 24 ⎞⎛ 26 ⎞P(E4|E1E2E3) = 1. An assignment is a sequence i1, ..., i20 where ij is the job to which person j is assigned. If A chooses (c) then B chooses (b). Since Y is normal with mean μ and variance σ2 it follows that its moment generating function is M(t) = E[etY] = e μt +σ 2 2 t /2 Hence, since X = eY, we have that E[X] = M(1) = e μ +σ 2 /2 and E[X 2] = M(2) = e 2 μ + 2σ 2 Therefore, 2 2 2 2 Var(X) = e2 μ + 2σ − e2 μ +σ = e 2 μ +σ (eσ − 1) 50. − Pi (λ pi) ri ri ! Throw a needle on a table, ruled with equidistant parallel lines a distance D apart, a large 2L number of times. E[XY] = E[E[XY|X]] = E[XE[Y|X]] Hence, if E[Y|X] = E[Y], then E[XY] = E[X]E[Y]. Pj Pj + P1 Similar to (b) of 19. FY(x) = P{eX ≤ x} = P{X ≤ log x} FX(log x) fY(x) = fX(log x)/x = 76 1 x 2π σ e − (log x − μ) 2 / 2σ 2 Chapter 5 Chapter 6 Problems 2. i n ∑X i i =1 ⎥ ⎥ / n − 75 > 5⎬ ≈ P{⎢Z⎢ > ⎥ n ⎥ ≤ .1 when n = 3 ⎛ 20 ⎞ (a) P⎨ X i > 15} ≈ 20 / 15 ⎨ i =1 ⎬ ⎛ 20 ⎞ ⎛ 20 ⎞ (b) P ⎨ X i > 15} = P ⎨ X i > 15.5⎬ ⎨ i =1 ⎬ ⎛ i =1 ⎬ ⎛ 15.5 − 20 ⎞ ≈ P⎨Z > ⎬ 20 ⎠ ⎭ = P{Z > −1.006} ≈ .8428 ∑ 5. Let 10 X= ∑ 10 Xi , Y = ∑ 10 Yi , W = i =1 E[X] = i =1 10 (a) ∑W ∑ E[X] = 10(3/10) 2 i = .9 i =1 10 (b) E[Y] = ∑ E[Y] = 10(7/10) 2 i = 4.9 i =1 (c) Since X + Y + W = 10, we obtain from parts (a) and (b) that E[W] = 10 − .9 − 4.9 = 4.2 Of course, we could have obtained E[W] from 10 E[W] = ∑ E[W] = 10(2)(3/10)(7/10) = 4.2 i i =1 9. (a) EFcGc j =1 (b) EFcG (c) E ∪ F ∪ G (d) EF ∪ EG ∪ FG (e) EFG (f) EcFcGc (g) EcFcGc ∪ EFcGc ∪ EcFGc ∪ EcFcG (h) (EFG)c (i) EFGc ∪ EFcG ∪ EcFG (j) S 7. 3 3 Also, 25 130 1 ⎛ ⎞ > a} ≤ = when a = P⎨Y − 2 3 10 ⎛ ⎞ 130 + 9a 1170 . Hence, Buffy is cheerful 3000/59 percent of the time. In both cases the one black ball is equally likely to be in either of the 4 positions. More formally, with L3 being the event that outcome 3 is the last to occur P(F1|L3) = P(L3 F1) P (F1) P (L3) = (1/ 2)(1/ 3) = 1/ 2 1/ 3 (b) With S1 being the event that the second trial results in outcome 1, we have P(F1S1|L3) = 78. (a) Simulate Xi from Fi, i = 1, 2. The inverse equation F−1(U) = X is equivalent to or β 1 − e −αX = U X = {−log(1 − U)/α}1/β Since 1 − U has the same distribution as U we can generate from F by generating a random number U and setting X = {−log(U)/α}1/β. Then E[Xi,j] = P{i, j is a matched pair} = 1 n(n − 1) Hence, the expected number of matched pairs is ⎡ ⎤ E⎢ X i, j ⎥ = ⎣⎦⎦ i < j ∑ 16. The distribution function is given by F(x) = x 2 / 4 − x + 1, 2 ≤ x ≤ 3, x − x 2 / 12 − 2, 3 ≤ x ≤ 6 Hence, for u ≤ 1/4, F−1(u) is the solution of x2/4 − x + 1 = u that falls in the region 2 ≤ x ≤ 3. 57. = 7 3 142 2 2 (.7) + (.2) 10 10 (b) P(J3 guilty|one of J1, J2 votes guilty} 7 3 (.7)2(.7)(.3) + (2.)2(.2)(.8) 15 10 . fR,θ(r,θ) = r, 0 < r sin θ < 1, 0 < r cos θ < 1, 0 < θ < π/2, 0 < r 1 ∞ fV(v) = 1 ∫ 2vu 2 du = v 1 ,v>1 2v 2 For v < 1 ∞ fV(v) = 1/ 2 Chapter 6 1 ∫ 2vu 2 du = 1 , 0 < v < 1. Then, E[g(Zn)] = ∫ x − c ≤δ g (x) dFn (x) + ∫ x − c >δ g (x)dFn (x) ≤ (ε + g(c))P{⎢Zn − c⎢ ≤ δ} + BP{⎢Zn − c⎢ > δ} In addition, the same equality yields that E[g(Zn)] ≥ (g(c) − ε)P{⎢Zn − c⎢ ≤ δ} − BP{⎢Zn − c⎢ > δ} Upon letting n → ∞ , we obtain that lim sup E[g(Zn)] ≤ g(c) + ε lim inf E[g(Zn)] ≥ g(c) − ε The result now follows since ε is arbitrary. ∫ fY(y) = c (y 2 − x 2)e − y dx −y = 4 3 −y cy e , −0 < y < ∞ 3 ∞ ∫ fY (y)dy = 1 ⇒ c= 1/8 and so fY(y) = 0 y 3e − y ,0 1/2|X < 1/2} = P{Y > 1/2, X < 1/2}/P{X < 1/2} 2 1/ 2 = ⎛ ∫ ∫⎝ x 2 + 1/ 2 0 1/ 2 ∫ (2 x 2 xy ⎞dxdy ⎢2 ⎠+ x) dx 0 10. Then, with Xi equal to 1 if item i is in the randomly chosen box⎡ 101 ⎤E[X] = E⎢ X i ⎥ = ⎣ i =1 ⎦∑ 101 101 ∑ E[X] = 10 i > 10 i =1 Hence, X can exceed 10, showing that at least one of the boxes must contain more than 10 items. (c) there are n(n − 1)(n − 2)2n − 3 selections in which the chair, secretary and treasurer are all different. Let Ji denote the indicator variable for the event that you will eventually be correct when guessing card i; and note that this event will occur if among cards 1 thru i, card 1 is first , card 2 is second, ..., and card i is the last among these i cards. Let the state be the number of pairs of shoes at the door he leaves from in the morning. If use (a) will win with probability p. Let A be the event that none of the final 3 balls were ever used and let Bi denote the event that i of the first 3 balls chosen had previously been used. μ= n ∑p i n by letting Number = i =1 ∑X i i =1 ⎧1 i is success where X i = ⎨ ⎩0 - - - n Var(Number) = ∑ p (1 − p) i i i =1 maximization of variance occur when pi ≡ μ/n minimization of variance when pi = 1, i = 1, ..., [μ], p[μ]+1 = μ − [μ] To prove the maximization result, suppose that 2 of the pi are unequal—say pi ≠ pj. (a) ∫e −x −x ∫e −x 0 ∞ (b) ∞ (c) ∫ 0 P{3,0} = P{0} e dx = 0 1 2 ∞ 1 x3 − x Γ(4) 1 e dx = e − y y 3dy = = 3! 96 0 96 16 ∫ x3 − x e dx 3! e− xe− x ∞ 23 33 + .4e − 3e − 3 3! 3! −2 −3 .6e + .4e .6 e − 2 e − 2 ∫e −x −x e dx = 2 2 = 4 81 3 0 1 70. Assume n > 1. The final step of parts (a) and (b) use that −Z is also a standard normal random variable. 1 n +1 Let X denote the number of items in a randomly chosen box. Condition on when the first tail occurs. Then in the 2 variance formula, we must show p + pj ⎞⎛ pi + p j ⎞ ⎥⎥1 − i ⎥≥ pi(1 − pi) + pj(1 − pj) 2⎥⎥ 2 ⎥⎝ 2 ⎠ or equivalently, pi2 + p 2j − 2 pi p j = (pi − p j) 2 ≥ 0. P(S|Cc) ≈ .9835. Then P(N1 ∪ N2) = P(N1) + P(N2) − P(N1N2) = (1 − p1)n + (1 − p2)n − (1 − p1 − p2)n Hence, the probability that both outcomes occur at least once is 1 − (1 − p1)n − (1 − p2)n + (p0)n. Then E[X] = E[X|G]P(G) + E[X|B]P(B) = 3(.4) + 5(.6) = 4.2 If Y is Poisson with mean λ, then E[Y 2] = λ + λ2. E[X] = ∞ ∑ 2 (1/ 2) n n =∞ n =1 (a) probably not (b) yes, if you could play an arbitrarily large number of games 31. (a) P{g − g|at least one g } = 1/ 4 = 1/3. There are 4! possible arrangements. E[Y] = E[X/σ − μ/σ] = 1 σ E[X] − μ/σ = μ/σ − μ/σ = 0 Var(Y) = (1/σ)2 Var(X) = σ2/σ2 = 1. The number i represents which wife is carrying the kitten, j then represents which of that wife's 7 sacks contain the kitten; k represents which of the 7 cats in sack j of wife i is the mother of the kitten; and l represents the number of the kitten of cat k in sack j of wife i. Since it rains today when the state is either 0 or 2 the probability is 2/5. FG = {(1, 4), (4, 1)}. (a) There will be a string of k consecutive heads within the first n trials either if there is one within the first n − 1 trials, or if the first such string occurs at trial n; the latter case is equivalent to the conditions of 2.⎡ 10 ⎤E⎢ Xi ⎥ = ⎣ i =1 ⎦∑ 1 4 x dx = 3 3 −3 / 2 ∫ 3/ 2 ∫ xdx = 3 / 2 . Player B. 14. 2 P{ X > 25 5 / 30 , P{X > 25⎢ X > 15} = = = 1/3 3 P{ X > 15} 15 / 30 where X is uniform (0, 30). P{A = superior ⎢ A fair, B poor} P{ A fair, B poor A superior A superior} = P{ A superior} = P{ A fair, B poor} 10 15 1 3 30 30 2 = = . (a) number of nonnegative integer solutions of x1 + x2 + x3 + x4 = 8. 12 9 , P(weak|Rc) = 35 35 Let M, T, W, Th, F be the events that the mail is received on that day. (a) After stage k the algorithm has generated a random permutation of 1, 2, ..., k. (ii) Since, P{trial i is a success} = 1/(2n − 1) it follows that, when n is large, Mk is approximately Poisson distributed with mean k/(2n − 1). Hence, the number of such couples is approximately Poisson with mean 80,000/365 ≈ 219.18. 134 Chapter 8 13. ∫ (a − x) f (x)dx + ∫ (x − a) f (x)dx E[|X − a|] = xa = aF(a) − ∫ xf (x)dx + ∫ xf (x)dx − a[1 − F (a)] xa Differentiating the above yields derivative = 2af(a) + 2F(a) − af(a) − af(a) − 1 Setting equal to 0 yields that 2F(a) = 1 which establishes the result. 10. 10 Chapter 2 10. Hence, since he is equally likely to choose either door when he leaves tomorrow it follows that Pi,i = Pi,5−i = Pi,i−1 = Pi,6−i = 1/4 provided all the states i, 5 − i, i − 1, 6 − i are distinct. P{X = n + k|X > n} = P{ X = n + k} P{ X > n} p(1 − p) n + k −1 (1 − p) n = p(1 − p)k−1 = If the first n trials are fall failures, then it is as if we are beginning anew at that time. (a) P{Smith blue gene} = P{X = 1|X ≤ 1} = 1/ 2 = 2/3 1 − 1/ 4 (b) Condition on whether Smith has a blue-eyed gene. P(A) = P(B) = P(C) = 1/2, P(AB) = P(AC) = P(BC) = 1/4. If use strategy (b) then P{win} = P{win|both correct}p2 + P{win|exactly 1 correct}2p(1 − p) + P{win|neither correct}(1 − p)2 2 = p + p(1 − p) + 0 = p Thus, both strategies give the same probability of winning. Therefore, on average, every four families will have one family with 2 sons and two families with 1 son. P(F) The result now follows since P(EF) = P(EFG) + P(EFGc) 27. That is, you will continually guess card 1 until it appears, and then card 2 until it appears, and so on. This gives 2 1 2n − 2 ⎛ 1 ⎞+ n Pn = n⎢ ⎢Pn −1 2 −1 2 −1 ⎝ 2 ⎠2 ⎛1⎞where⎢ ⎢is the probability they both win given they do not play each other. P1,1 = 1/2. Similarly, let Yj equal 1 if the jth blue ball is selected and let it be 0 otherwise. You can download the paper by clicking the button above. (a) The probability that an arbitrary couple were both born on April 30 is, assuming independence and an equal chance of having being born on any given date, (1/365)2. Then, P(Fj|E) = = = P(EFj) P(E) P(Fj) P(E Fj) ∑ j P(Fj) P(E Fj) p j (1/ j) .1 + .25(1/ 2) + .35(1/ 3) + .3(1/ 4) = .24 Thus, P(F1|E) = .24, P(F4|E) = .18. (b) P(A) = .48(.38) + .37(.62) = .4118 Therefore, 41.18 percent of the class attended. Let N(t) denote the number after stage t. Let Y = log(X). 509, Solution to Problem 68 of Chapter 4: change (1 - e −5)80 to (1 - e −5)10 Hence, generate random numbers U1 and U2 and set X = U1 if U2 ≤ 3125U13 (1 − U1) 2 / 108 . Pn = Pn−1p + (1 − Pn−1)(1 − p) = (2p − 1)Pn−1 + (1 − p)⎡1 1 ⎤ = (2 p − 1)⎢ + (2 p − 1) n − 1 ⎥ + 1 − p by the induction hypothesis⎣2 2 ⎦2 p −1 1 = + (2 p − 1) n + 1 − p 2 2 1 1 = + (2 p − 1) n . P{child blue} = P{blue|blue gene}(2/3) + P{blue|no blue}(1/3) = (1/2)(2/3) = 1/3 (c) First compute P{Smith blue|child brown} = P{child brown Smith blue}2/3 2/3 = 1/2 Now condition on whether Smith has a blue gene given that first child has brown eyes. n −1 3 · 9!+7 · 6 · 3 · 7!+7 · 6 · 5 · 4 · 3 · 5!+7 · 6 · 5 · 4 · 3 · 2 · 3 · 3! 10! ⎛ 5⎞⎛ 6⎞⎛8⎞ ⎢ ⎥+⎢ ⎥+⎢ ⎥3 3 3 P{same} = ⎝ ⎠⎝ ⎠⎝ ⎠⎛19 ⎞⎢ ⎥⎝3⎠⎛ 5 ⎞⎛ 6 ⎞⎛ 8 ⎞⎛19 ⎞P{different} = ⎢ ⎥⎢ ⎥⎢ ⎥⎢ ⎥⎝ 1 ⎠⎝ 1 ⎠⎝ 1 ⎠⎝ 3 ⎠If sampling is with replacement P{same} = 53 + 63 + 83 (19)3 P{different} = P(RBG) + P{BRG) + P(RGB) + ... + P(GBR) 6 ·5· 6 ·8 = (19)3 Chapter 2 13 29. (b) P(A wins) = 85. If Ai is the event that couple number i are seated next to each other, then these events are, when n is large, roughly independent. (b) there are 3n(n − 1)2n − 2 selection in which the chair, secretary and treasurer jobs are held by 2 people. This follows immediately from Chebyshev's inequality. (a) and (b): (i) i =5 7 (iii) ⎛7⎞∑⎜ ⎝ i ⎟⎠p (1 − p) i ⎛9⎞∑⎜ ⎝ i ⎟⎠p (1 − p) 7 −i i 9 −i 8 , (ii) ⎛8⎞∑⎜ ⎝ i ⎟⎠p (1 − p) i 8−i , i =5 where p = .7 in (a) and p = .3 in (b). (a) E[X] since whereas the bus driver selected is equally likely to be from any of the 4 buses, the student selected is more likely to have come from a bus carrying a large number of students. On the other hand, the chairperson can be anyone of k k =1 ⎝ ⎠∑ the n persons and then each of the other n − 1 can either be on or off the committee. To determine E[X], arbitrarily number the 8 failed components and let, for i = 1, ..., 8, ⎧1, if failed component i is among the group of 12 components Xi = ⎨ ⎩0, otherwise Then, 8 X= ∑X i i =1 and so 8 E[X] = ∑ E[X] i i =1 Chapter 7 105 Because Xi will equal 1 if the randomly selected component is either failed component number i or any of its 11 neighboring components in the counterclockwise direction, it follows that E[Xi] = 12/47. ∑ The number of stages is a negative binomial random variable with parameters m and 1 − p. there are at present i distinct types. (a) P{Z > x} = P{−Z < −x} = P{Z < −x} (b) P{⎢Z⎢ > x} = P{Z > x} + P{Z < −x} = P{Z > x} + P{−Z > x} = 2P{Z > x} (c) P{⎢Z⎢< x} = 1 − P{⎢Z⎢ > x} = 1 − 2P{Z > x} by (b) = 1 − 2(1 − P{Z < x}) 10. 33. (a) P{goes to A} = P{5 < X < 15 or 20 < X < 30 or 35 < X < 45 or 50 < X < 60}. Choose a random member of the class. (a) Let X1 denote the number of nonspades preceding the first ace and X2 the number of nonspades between the first 2 aces. (a) P{X1 > X2|X1 > X3} = P{ X 1 = max(X 1 , X 2 , X 3)} 1 / 3 = 2/3 = P{ X 1 > X 3} 1/ 2 (b) P{X1 > X2|X1 < X3} = P{ X 3 > X 1 > X 2 } 1 / 3! = = 1/3 1/ 2 P{ X 1 < X 3} (c) P{X1 > X2|X2 > X3} = P{ X 1 > X 2 > X 3} 1 / 3! = = 1/3 P{ X 2 > X 3} 1/ 2 (d) P{X1 > X2|X2 < X3} = P{ X 2 = min(X 1 , X 2 , X 3)} 1 / 3 = 2/3 = P{ X 2 < X 3} 1/ 2 P{U > s|U > a} = P{U > s}/P{U > a} 1− s = ,a max X kj , X ij = min X ik ⎬ k k ≠i ⎣k =1,..., m ⎦ ⎨ ⎬ { = P min X ik > max X kj P X ij = min X ik k k ≠i k } where the last equality follows as the events that every element in the ith row is greater than all elements in the jth column excluding Xij is clearly independent of the event that Xij is the smallest element in row i. Thus, P(heads) = P(H, T⎢H, T or T, H) P(H ,T) p (1 − p) 1 = = = P(H , T) + P (T , H) p(1 − p) + (1 − p) p 2 No, with this new procedure the result will be heads (tails) whenever the first flip is tails (heads). If Player A chooses spinner (a) then B can choose spinner (c). Since only one person can be assigned to a job, it follows that the sequence is a permutation of the numbers 1, ..., 20 and so there are 20! different possible assignments. For (c) E[XY] = E[E[XY|X]] = E[XE[Y|X]]⎡ ⎛ σy ⎞⎤ = E⎢ X ⎢⎣ μ y + ρ (X − μ x ⎢⎥σx ⎠⎦⎥⎦ ⎝ = μxμ y − ρ σy 2 σ μ x + ρ y μ x2 + σ x2 σx σx ( ) and so Corr(X, Y) = Chapter 7 ρσ yσ x =ρ σ yσ x 129 41. 2 (a) 2n − 1 2 (b) 2n − 2 (c) exp{−2n/(2n − 1)} ≈ e−1 67. f R1 (r ,θ) = r π = 2r 1 , 0 ≤ r ≤ 1, 0 ≤ θ < 2π. Then P{f(X) > a} = P{X > f −1(a)} ≥ P{Y > f −1(a)} since x ≥st Y = P{f(Y) > a} Therefore, f(X) ≥st f(Y) and so, from Exercise 7, E[f(X)] ≥ E[f(Y)]. Let Xi denote the number of type i events, i= 1, ..., n. 0 (a) Use Exercise 6 to obtain that E[X] = ∞ ∞ 0 0 ∫ P{X > t}dt ≥ ∫ P{Y > t}dt = E[Y] (b) It is easy to verify that X+ ≥st Y+ and Y− ≥ st X− Now use part (a). ⎛n⎞Number of possible selections of a committee of size k and a chairperson is k⎢ ⎥and so ⎝k⎠n ⎛n⎞k⎢ ⎥represents the desired number. 333 9 P{divisible by 105} = 1000 1000 142 P{divisible by 7} = 1000 66 P{divisible by 15} = 1000 (a) P{divisible by 3} = In limiting cases, probabilities converge to 1/3, 1/7, 1/15, 1/10 (b) P{μ(N) ≠ 0} = P{N is not divisible by pi2 , i ≥ 1} ∏ P{N is not divisible by = ∏ (1 − 1 / p) = 6/π = pi2 } i 2 i 2 i 13. (a) .6e−2 + .4e−3 (b) .6e−2 + .4e−3 23 33 + .4 e − 3 3! 3! (b) .6e−2 (c) P{3⎢0} = ∞ 69. (a) n(n − 1) + m(m − 1) (n + m)(n + m − 1) (b) Putting all terms over the common denominator (n + m)2(n + m − 1) shows that we must prove that n2(n + m − 1) + m2(n + m − 1) ≥ n(n − 1)(n + m) + m(m − 1)(n + m) which is immediate upon multiplying through and simplifying. Hence n ⎛ n⎞there are j = k⎢ ⎥vectors that meet the criterion. E[Number] = 2 3⎡ 1 2 ⎤17 . Then, P(A ∪ B) = P(A) + P(B) − P(AB) 4 · 4 · 16 4 · 4 · 16 · 3 · 15 + = 52 · 51 52 · 51 · 50 · 49 = .0983 where the preceding used that P(A) = P(B) = 2 × 4 · 16 . (c) μx = E[X] = E[Y + Z] = μ σ x2 = Var(X) = Var(Y + Z) =Var(Y) + Var(Z) = σ2 + 1 Cov(Y + Z , Y) σ ρ = Corr(X, Y) = σ σ 2 +1 = σ 2 +1 (d) and (e) The conditional distribution of Y given X = x is normal with mean E[Y|X = x] = μ + ρ σ σ2 (x − μx) = μ + (x − μ) 1+σ 2 σx and variance ⎛ σ2 ⎞σ2 Var(Y|X = x) = σ 2 ⎢1 − 2 ⎥= 2 ⎝ σ +1⎠σ +1 132 Chapter 7 Chapter 8 Problems 1. 48. Hence, E[Return of x-strategy] = B (A + B) / 2 if A < x < B otherwise (c) This follows from (b) since there is a positive probability that X will lie between A and B. 5 5 13. ⎝ k ⎠⎝ k − 1 ⎠⎝ k − 1⎠⎝ k ⎠Chapter 2 19 Chapter 3 Problems 1. ⎢good}P{g} + P{acc.|ave.}P{ave.} + P{acc.|bad P(b)} = (.05)(.2) + (.15)(.5) + (.30)(.3) = .175 .95(2) .825 (.85)(.5) P{A is average|no accident} = .825 P{A is good⎢ no accident} = 51. Let E be the event that a randomly chosen pregnant women has an ectopic pregnancy and S the event that the chosen person is a smoker. ⎛ 35 ⎞1−⎢ ⎥⎝ 36 ⎠43. Assume that the joint density factors as shown, and let Ci = ∫ ∞ −∞ gi (x)dx, i = 1, ..., n Since the n-fold integral of the joint density function is equal to 1, we obtain that n 1= ∏C i i =1 Integrating the joint density over all xi except xj gives that ∏C f X j (x j) = g j (x j) Chapter 6 i = g j (x j) / C j i≠ j 91 If follows from the preceding that n f(x1, ..., xn) = ∏f j =1 Xj (x j) which shows that the random variables are independent. Let Y = max (X1, ..., Xn) , Z = min(X1, ..., Xn) P{Y ≤ x} = P{Xi ≤ x, i= 1, ..., n} = n ∏ P{ X ≤ x} = F n (x) i 1 n P{Z > x} = P{Xi > x, i = 1, ..., n} = ∏ P{ X i > x} = [1 − F (x)]n . (3) + ⎢2 + + 2 ⎥ = 5 5⎣ 3 3⎦5 The above assumes that when 3 remain, you choose 1 of the 3 and ask if that is the one. ∫ xe −x / 2 dx = −2 xe − x / 2 − 4e − x / 2 . Then, with p(i, j) = P{X = i, Y = j} 6 ∑ E[return] = 2 jp (1, j) + j =1 = 2. Let f denote for and a against a certain place of legislature. No. Let Xi = ⎨⎧1 if trial i is a success . P1 = 1/3 P2 = (1/3)(4/5) + (2/3)(1/5) = 2/5 P3 = (1/3)(4/5)(6/7) + (2/3)(4/5)(1/7) + (1/3)(1/5)(1/7) = 3/7 P4 = 4/9 (b) Pn = 2n + 1 (c) Condition on the result of trial n to obtain Pn = (1 − Pn−1) 1 2n + Pn −1 2n + 1 2n + 1 (d) Must show that n n −1 ⎡ 1 n − 1 2n⎡ = 1− + 2n + 1 ⎢ 2n − 1 ⎦2n + 1 2n − 1 2n + 1 2n + 1 2n + 1 or equivalently, that n n 1 n − 1 2n = + 2n + 1 2n − 1 2n + 1 2n + 1 2n − 1 2n + 1 But the right hand side is equal to n + 2n(n − 1) n = (2n − 1)(2n + 1) 2n + 1 42 Chapter 3 18. E[X2] = [(40)3 + (33)3 + (25)3 + (50)3]/148 ≈ 1625.4 Var(X = E[X2] − (E[X])2 ≈ 82.2 E[Y2] = = [(40)2 + (33)2 + (25)2 + (50)2]/4 = 1453.5, 38. Use the notation in Problem 9, n X= ∑X j j =1 where Xj is 1 if box j is empty and 0 otherwise. Suppose n components are in stock. Chapter 5 28. should be 84. a x − b⎞ ⎛ x−b⎞When a< 0, FY(x) = P ⎨ X ≥ ⎢ and so ⎬ = 1 − FX⎢ a⎠ ⎝ ⎝ a ⎠1 ⎛ x−b⎞fY(x) = − f X⎢ + X n ⎠E⎢ f⎢ 1 ⎥= n ⎝ ⎠⎝ ⎛n⎞∑ f (k / n)⎢ k ⎟x (1 − x) k n−k k =1 The result now follows from Exercise 4. 102 n ∑1/ i! i =1⎡ 52⎤1 match on card i E[number of matches] = E⎢ I i ⎥, I i = ⎨ ⎩0 - -⎣ i =1 ⎦⎦ = 52 = 4 since E[Ii] = 1/13 13 ∑ Chapter 7 19. c (a) A = ∪Ai, (b) yes P (Ai) = n(1/2)n−1 (c) P(A) = ∑ 17. ⎛ N1 + N 2 − k ⎞⎛ N + N2 − k ⎞N + N −k N + N −k⎢ ⎥(1 / 2) 1 2 (1 / 2) + ⎡ 1 ⎥(1 / 2) 1 2 (1 / 2) N N 1 2 ⎝ ⎠⎝ ⎠77. I 13. Hence, the probability that T, H, H, H occurs first is 15/16. αn+1 = αnp + (1 − αn)(1 − p1) Pn = αnp + (1 − αn)p1 21. S = {(n, x1, ..., xn−1), n ≥ 1, xi ≠ 6, i = 1, ..., n − 1}, with the interpretation that the outcome is (n, x1, ..., xn−1) if the first 6 appears on roll n, and xi appears on roll, i = 1, ..., n − 1. Let X and Y denoted respectively the locations of the ambulance and the accident of the moment the accident occurs.⎡ 1 = E⎢ ⎣ n ∑ Xi 1 1 ⎤Xi ⎥ = ⎦ n ∑ 1⎡ E⎢Xi ⎣ n ∑ 1 ⎤⎡ X i ⎥ = nE⎢ X 1 ⎦⎣ ⎤n ∑ X ⎦⎦ i 1 Hence,⎡ E⎢ ⎣ 11. The first gift can go to any of the 10 children, the second to any of the remaining 9 children, and so on. (a) P{Y1 = 1|Y2 = 1} = p(1, 1)/[1 − (12/13)3] = 1 − P{Y1 = 0|Y2 = 1} (b) P{Y1 = 1|Y2 = 0} = p(1, 0)/(12/13)3 = 1 − P{Y1 = 0|Y2 = 0} where p(1, 1) and p(1, 0) are given in the solution to Problem 5. (a) 25 = 32 (b) W = {(1, 1, 1, 1, 1), (1, 1, 1, 1, 0), (1, 1, 1, 0, 1), (1, 1, 0, 1, 1), (1, 1, 1, 0, 0), (1, 1, 0, 1, 0) (1, 1, 0, 0, 1), (1, 1, 0, 0, 0), (1, 0, 1, 1, 1), (0, 1, 1, 1, 1), (1, 0, 1, 1, 0), (0, 1, 1, 1, 0), (0, 0, 1, 1, 1), (0, 0, 1, 1, 0), (1, 0, 1, 0, 1), (1, 0, 1, 0, 0)} (c) 8 (d) AW = {(1, 1, 1, 0, 0), (1, 1, 0, 0, 0)} 6. (a) 615 (b) 615 − 315 (c) 415 8. Let μ = E[X]. 2 = 1/3 since X is uniform (0, 60). Let N go to infinity in Example 4j. P(W|F) = 55. 64 = 1296 3. With g(x) = λe−λx 2 f (x) 2e − x / 2 2 = exp{−[(x − λ) 2 − λ2] / 2} g (x) λ (2π)1 / 2 e− λx λ (2π)1 / 2 2 2e λ / 2 exp{−(x − λ) 2 / 2} 1/ 2 λ (2π) Hence, c = 2eλ 2 /2 /[λ (2π)1 / 2] and simple calculus shows that this is minimized when λ = 1. As⎢⎥ n1 ,..., ni − 1,...nr ⎥⎥represents the number of elements 1, 2, ..., n where n = i =1 ⎝ ⎠possibilities when person n is put in subset i, the result follows. ⎛ 94 ⎞⎢ ⎥10 (a) P{X = 0} = ⎝ ⎠⎛100 ⎞⎢ ⎥⎝ 10 ⎠⎛ 94 ⎞⎛ 94 ⎞ 6 ⎞⎛ 94 ⎞ 6 ⎞⎢ ⎢+⎢ ⎢ ⎢+⎢ ⎢10 9 1 8 2 (b) P{X > 2} = 1 − ⎝ ⎠⎝ ⎠⎝ ⎠⎝ ⎠⎛100 ⎞⎢ ⎥⎝ 10 ⎠80. 59. P(B|A) = 1, P(B|Ac) = P(BAc) P(Ac) Let F be the event that a first born is chosen. = 19 18 ⎛ Var⎢ ⎢⎝ Chapter 7⎡ 2 2 ⎛ 2 ⎞2 ⎤⎛2⎞⎛ 2⎞X j ⎥ = 10⎢1 − ⎥+ 10 · 9⎢ −⎢ ⎥⎥19 19 19 18 ⎝ 19 ⎠⎥⎦ j =1 ⎥⎦ 10 ∑ 107 35. + xr) n − i1 by the Binomial theorem ... (a) E(N) = 2[p2 + (1 − p)2] + 3[2p(1 − p)] = 2 + 2p(1 − p) The final equality could also have been obtained by using that N = 2 +] where I is 0 if two games are played and 1 if three are played. (a) P(Ei) = 1 − ∑ 2 j =0 ⎛ 365 ⎞j 365 − j⎢ ⎥(1/ 365) (364 / 365) ⎝ j ⎠(b) exp(−365P(E1)) 24. Let Ni denote the event that none of the trials result in outcome i, i = 1, 2. ⎛ n ⎞⎛ m ⎞ ⎢ ⎢(n − r + 1) ⎝ r − 1 ⎠⎝ k − r ⎠⎛ n + m⎞⎢ ⎢(n + m − k + 1) ⎝ k − 1 ⎠Let y1, y2, ..., yk denote the successive runs of losses and x1, ..., xk the successive runs of wins. E[X n] = ∞ ∑ i e λ λ / i! i =0 ∞ = ∑i n − i n −1 − λ i e λ /(i − 1)! i =1 ∞ = i ∑ i e λ λ / i! i =1 ∞ = n − ∑ (j + 1) n −1 − λ e λ j +1 / j ! j =0 = λ ∞ ∑ (j + 1) n −1 − λ e λ j / j! j =0 = λE[(X + 1) n −1] Hence [X 3] = λE(X + 1)2] =λ ∞ ∑ (i + 1) 2 e λ λ / i! 2 − i i =0 ∞ ∞ ⎡∞ ⎤ = λ⎢ i 2e − λ λi / i ! +2 ie − λ λi / i ! + e − λ λi / i ! ⎥i =0 i =0 ⎣ i =0 ⎦ 2 = λ[E[X] + 2 E[X] + 1) = λ(Var(X) = E2[X] + 2E[X] + 1) = λ(λ + λ2 + 2λ + 1) = λ(λ2 + 3λ + 1) ∑ 20. (b) 0. P{A|failure} = 43. As a similar result is true, when B is first, we see that the probability in this case is 2 · 3 · 3!/5! = 3/10 (b) 2 · 2 · 3!/5! = 1/5 (c) 2 · 3!/5! = 1/10 Chapter 2 15 (n − 1) k −1 if do not discard nk 45. 3 18. (a) P{X ≥ 50} = P{X ≥ 49.5} = P{X − 65}/4.77 ≥ −15.5/4.77 ≈ P{Z ≥ −3.25} ≈ .9994 (b) P{59.5 ≤ X ≤ 70.5} ≈ P{−5.5/4.77 ≤ Z ≤ 5.5/4.77} = 2P{Z ≤ 1.15} − 1 ≈ .75 (c) P{X ≤ 74.5} ≈ P{Z ≤ 9.5/4.77} ≈ .977 22. ⎛n⎞log P{X = k} = log⎢ ⎥+ k log p + (n − k) log (1 − p) ⎝k ⎠n−k ∂ log P{x = k} = − p 1− p ∂p = 0 ⇒ p = k/n maximizes 14. (b) Pn,1 = P{A receives first 2 votes} = n(n − 1) n − 1 = (n + 1)n n + 1 Pn, 2 = P{A receives first 2 and at least 1 of the next 2} n n −1 ⎡ 2 ·1 ⎤ n − 2 = {1 − ⎬= n + 2 n + 1 ⎣n(n − 1) ⎦ n + 2 (c) Pn,m = n−m , n ≥ m. (b)⎢ ⎥= 4 ⎝2, 2, 2, 2 ⎠(2) 29. ∑ Chapter 1 7 19. 16. 22. ⎝3⎠⎝ 3 ⎠⎝3⎠⎝ 1 ⎠⎝2 ⎠⎛8⎞⎝ 4⎠There are⎢ ⎥⎢that do not contain either of the 2 men, and there are ⎝ 3⎠⎝ 3 ⎠contain exactly 1 of them. Cov(X + Y, X − Y) = Cov(X, X) + Cov(X, −Y) + Cov(Y, X) + Cov(Y, −Y) = Var(X) − Cov(X, Y) + Cov(Y, X) − Var(Y) = Var(X) − Var(Y) = 0. P{5th|heads} = P{heads 5th }P{5th } ∑ P{h i th }P{i th } i 5 1 1 = 1010 10 = . Let Xj equal 1 if urn j is empty and 0 otherwise. (a) P{W ≤ t} = 1 − P{W > t} = 1 − P{X > t, Y > t} = 1 − [1 − FX(t)] [1 − FY(t)] (b) fW(t) = fX(t)[1 − FY(t)] + fY(t) [1 − FX(t)] Dividing by [1 − FX(t)][1 − FY(t)] now yields λW(t) = fX(t)/[1 − FX(t)] + fY(t)/[1 − FY(t)] = λX(t) + λY(t) 9. (a) E[X] = 6 (b) E[X|Y = 1] = 1 + 6 = 7 2 3 4 1 4 1 ⎛4⎞1 ⎛4⎞1 ⎛4⎞(c) 1 + 2 + 3⎢ ⎥+ 4⎢ ⎥ ⎥+⎢ ⎥ ⎥+ ⎥(5 + 6) 5 5 5 ⎝5⎠5 ⎝5⎠5 ⎝5⎠5 ⎝ 5 ⎠49. (a) (b) ç2 1 = 2 2 2 2 μ α r a λ = λ λ np np (1 − p) = np /(1 − p) (c) answer = 1 (d) 1/ 2 1 / 12 = 3 (e) answer = 1 (d) answer = μ / σ 4. 1 1 1000 + 250 = 625 2 2 3 7 3 11 = p − 3/ 4 , (b) − p + (1 − p)2 = − p + 2 4 4 4 4 7 11 p − 3 / 4 = − p + 2 ⇒ p = 11 / 18 , maximum value = 23.72 4 4 (a) p − (1 − p) 3 3 (d) − q + 2(1 − q) , minimax value = 23/72 (c) q − (1 − q) 4 4 attained when q = 11/18 25. P{next m heads|first n heads} = P{first n + m are heads}/P(first n heads} 1 1 n +1 n+m = p dp . 24. 17. Hence, 2 percent smoke cigars but not cigarettes. Then, P(B) = P(B|B1)P(B1) + P(B|B2)P(B2) = (1/2)(1/2) = (2/3)(1/2) = 7/12 P(W B1) P(B1) (1 / 2)(1 / 2) P(B1|W) = = = 3/5 5 / 12 P(W) 24 Chapter 3 31. ∑ n +1 2 i /n ~ i =1 2 2 Var(Y) ~ n ∫ 1 x 2 dx n 2 ~ n 3 2 n ⎛ n +1⎞n −⎢ ⎥~ 3⎝ 2 ⎠12 12 1 i ... ⎛ 6 ⎞ 6 ⎞⎛12 ⎞⎢ ⎢ ⎥⎢ ⎥⎢ ⎝ 3 ⎠⎝ 3 ⎠⎝ 6 ⎠50. and 74. k 6. Hence, θE[X] ≤ 0 and thus θ > 0. n nm + . The probability that a given contestant does not beat all the members of some given subset of k other contestants is, by independence, 1 − (1/2)k. g (b + g − 1)! g = (b + g) ! b+ g 33. ⎛ 30 ⎞⎛ 54 ⎞1 −⎢ ⎥⎢ ⎥≈ .8363 ⎝3⎠⎝3⎠14 Chapter 2 36. 4 , 3 M0,b = b, M3,1 = 7 , 4 Ma,b = Mb,a M3,2 = 3/2 Let Xn denote the number of white balls after the nth drawing E[Xn+1|Xn] = X n Xn X ⎞ 1 ⎞ + (X n + 1)⎢1 − n ⎥=⎢1 − ⎥Xn +1 a+b ⎝ a+b⎠⎝ a+b⎠Taking expectations now yields (a). (a) 1 − ∞ ∑αp n = 1− n =1 αp 1 − p (b) Condition on the number of children: For k > 0 ∞ P{k boys} = ∑ P{k n children}αp n =1 ∞ = ⎛n⎞∑⎜ k ⎟(1/ 2) αp n n n=k P{0 boys} = 1 − 17. 8. That is, we need that Y2 ≥ Y + 2. ⎝ i =1 ⎠∑ 106 Chapter 7 32. Chapter 3 37 (b) and (d) Let Pij (Pij) denote the probability that A wins when A needs i more and B needs j more and A(B) is to flip. Hence, ∞ ∫ c xe − x / 2 dx = 1 ⇒ c = 1 / 4 0 ∞ 1 11 xe− x / 2 dx = [10e −5 / 2 + 4e − 5 / 2] 45 4 ∫ P{X > 5} = = 3. (a) Chapter 7 ⎛100 ⎞ 1 ⎞⎛ 364 ⎞365⎢ ⎢ ⎢⎝ 3 ⎠⎝ 365 ⎠⎝ 365 ⎠97 103 ⎧1 if day j is someones birthday (b) Let Xj = ⎨ ⎩0 - -⎡ ⎛ 364 ⎞100 ⎡ 365 ⎤365 E⎢ X j ⎥ = E[X j] = 365⎢1 − ⎥ ⎥1 ⎣ 1 ⎦⎦ ⎝ 365 ⎠⎦⎦ ∑ ∑ 22. P(F1c F2) (.9)(.2) P(F2 F1c F2c F3c)c) = = = .3629 1 − .504 .496 13. ∑ ⎛n⎞k 2⎢ ⎥n−2 ⎝ k ⎠= 2 n(n + 1) E[X 2] = n 2n − 1 k =0 2 − 1 n ∑ Var(X) = E[X 2] − {E[X])2 = ~ 62 n 22 n − 2 − n(n + 1)2n − 2 (2n − 1) 2 n22 n − 2 n = 4 22 n Chapter 4 n +1 E[Y] = , E[Y 2] = 2 35. 2 E[X] = ∫ P{ X > x}2 x ∞ ∫ dx = 2 xe − λx dx = 2 −1 0 12. For this strategy, the ith guess will be correct with probability 1/(n − i + 1) and so n E[N] = ∑1/(n − i + 1) i =1 (c) Suppose you will guess in the order 1, 2, ..., n. Therefore, as the solution of the equations u = x, v = x/y is x= u, y = u/v, we see that fu,v(u, v) = 96 u v 2 f X ,Y (u , u / v) = u 1 − (u 2 + u 2 / v 2) / 2 e v 2 2π Chapter 6 Hence, fV(u) = = = = Chapter 6 2 2 1 ∞ u e − u (1+1 / v) / 2 du 2 −∞ 2πv 2 2 1 ∞ u e − u / 2σ du , where σ2 = v2/(1 + v2) 2 −∞ 2πv 1 ∞ − u 2 / 2σ 2 ue du πv 2 0 1 2 ∞ −y σ e dy 0 πv 2 1 π (1 + v 2) ∫ ∫ ∫ ∫ 97 Chapter 7 Problems 1. They can then be paired up ⎛ 5 ⎞ 5 ⎞in 5! ways, since if we arbitrarily order the men then the first man can be paired with any of ⎛10 ⎞⎛12 ⎞the 5 women, the next with any of the remaining 4, and so on. = 2 3 i +1 i +1 0) P(X < ∞} = lim P{ X ≤ i} (a) P{X > i} = i →∞ = lim(1 − 1 /(i + 1)) = 1 i ∑ iP{ X = i} = ∑ i (P{ X > i − 1} − P{ X > i} (c) E[X] = i i = ⎛1 1 ⎞∑ i⎢ i − i + 1 ⎥⎥i = 1 ∑ i +1 i =∞ Chapter 4 63 Chapter 5 Problems 1 1. By symmetry, E[Z 3] = E[Z] = 0 and so Cov(Z,Z 3) = 0. Conditioning on the outcome of the first two games gives P(A = P(A|a, a)p2 + P(A|a, b)p(1 − p) + P(A|b, a)(1 − p)p + P(A|b, b)(1 − p)2 = p2 + P(A)2p(1 − p) where the notation a, b means, for instance, that A wins the first and B wins the second game. 2 · 4 · 16 52 · 51 19. 3 · 2 ·1 2 = 3·3·3 9 3 1 = P{same} = 27 9 P{complete} = 32. Generate a random number U and then set X = F−1(U). 3 ⎥ ⎥ 20. fX|Y(x|y) = e− x / y e− y / y ∞ ∫e −x / y e − y / y dx 1 −x / y e , 0 1)P(N > 1) = F(x)p + F(x)P(M ≤ x)(1 − p) again giving the result P(M ≤ x) = Chapter 7 pF (x) 1 − (1 − p) F (x) 115 62. P{⎢Y − X⎢ < a} = P{X < Y < X + a} + P{X < Y < X + a} 2 = 2 L L min(y + a , L) ∫ ∫ dxdy 0 y L−a y +a L L ⎤2⎡⎥ ⎥dxdy dxdy + L2⎢ 0 y ⎢⎣ L−a y ⎣ L−a a a⎛ a⎞ =1− + 2 (L − a) ⎢ 2 − ⎥, 0 < a < L L L⎝ L⎠L ∫ ∫ = 15. Choose a customer at random. n N ⎛ ⎞ ⎥⎝n⎠N ⎝N⎠ ⎥ ⎣ n ≤ j ≤ N ⎝n⎠⎝ N ⎠ ⎥⎝n⎠⎝ j ⎠∑⎝ n ⎠⎝ j =n n N +1 ⎛ i − 1 ⎞⎢ ⎥⎝ N ⎠i = n +1 ⎝ n + 1 − 1⎠⎢ ⎥⎝n⎠∑ n ⎛ N + 1 ⎞⎢ ⎥⎝ N ⎠⎝ n +1 ⎠⎢ ⎥⎝n⎠n(N + 1) n +1 Let Y denote the largest of the remaining m chips. 40 Chapter 3 4. P(Ai,j) = 1/365. P{X = 1} = 1/2, P{X = 2} = 5. P{6 ⎪ sum of 7} = P{(6, 1)} 1 / 6 = 1/6 P{6 ⎪ sum of 8} = P{(6, 2)} 5 / 36 = 1/5 P{6 ⎪ sum of 9} = P{(6, 3)} 4 / 36 = 1/4 P{6 ⎪ sum of 10} = P{(6, 4)} 3 / 36 = 1/3 P{6 ⎪ sum of 11} = P{(6, 5)} 2 / 36 = 1/2 P{6 ⎪ sum of 12} = 1. Chapter 3 39 Theoretical Exercises 1. Conditioning on which is the case gives the following. Let X be the number of storms, and let G(B) be the events that it is a good (bad) year. P{all white} = 1⎡ 5 5 4 5 4 3 5 4 3 2 5 4 3 2 1⎤ + + + + ⎥ 6⎣15 15 14 15 14 13 15 14 13 12 15 14 13 12 11 ⎦⎦1 5 4 3 P{3⎢all white} = 6 15 14 13 P{all white} 48. (b) Let ⎧1 a type j is caught before a type 1 Xj = ⎨ ⎩0 otherwise. 0 10 ∑ E[X] = 10(7/2) = 35. The f X (a) = 1 , 0 < a < 1 That is, ⎢X⎢ is uniform on (0, 1). We use Equation (6.3). 1 8 + (1 − PB) 9 9 Similarly, PB = 5 31 + (1 − PA) 36 36 Solving these equations gives that PA = 9/19 (and that PB = 45/76.) 75. P(M|C) = P(C M) P(M) c c P(C M) P(M) + P(C M) P(M) = (.05) p (.05) p + (.0025)(1 − p) 27. If not, repeat. ∫∫ ∫ daf (x, y)dydx ∫∫ g (x, y)dydx (X − μ)2 +... 2 (X − μ)2 ≈ g(μ) + g′(μ)(X − μ) + g′′(μ) 2 g(X) = g(μ) + g′(μ)(X − μ) + g′′(μ) Now take expectations of both sides. Hence, the probability that at least one person has the same birthday as you is approximately 1 − e−n/365. (c) In this case, ⎛ ⎞90 − 80 P(X1 + X2 > 90) = P⎢ Z > ⎥⎥ 72 + 2[.2(6)(6)] ⎦⎦⎝ = P(Z > 1.076) ≈ .141 Chapter 7 121 Theoretical Exercises 1. ⎝ 14 ⎠14 (c) Starting from the end of the deck the expected position of the first (from the end) heart is, 53 . 31. ⎛13 ⎞ 39 ⎞ 8 ⎞ 31⎞⎛ 52 ⎞ 39 ⎞⎢ ⎢ ⎢ ⎢ ⎢5 ⎝ 8 ⎠⎝ 5 ⎠⎝ 13 ⎠⎝ 13 ⎠51. E[NiNj⎢Ni] = NiE[Nj⎢Ni] = Ni(n − Ni) pj since each of the n − Ni trials no resulting in 1 − pi outcome i will independently result in j with probability pj/(1 − pi). If they are not then the probabilities are added. (a) P{X1 = 1⎢X2 = 1} = 5/13 = 1 − P{X1 = 0⎢X2 = 1} (b) same as in (a) 37. n (c) ⎛n⎞∑ ⎝ i ⎠p (1 − p) i n −i i =k Chapter 3 33 68. P(G|C) = 35. If E[Y|X] is linear in X, then it is the best linear predictor of Y with respect to X. Then the problem states that P(E|S) = 2P(E|Sc), P(S) = .32 Hence, P(S|E) = P(SE)/P(E) P(E S) P(S) = P(E S) P(S) + P(E S c) P(S C) 2 P(S) 2 P(S) + P(S c) = 32/66 ≈ .4548 = 16. 1 since each of the 3 points is equally likely to be the middle one. k =i (c) No. 84 Chapter 6 40. The Markov chain requires 4 states: 0 = RR = Rain today and rain yesterday 1 = RD = Dry today, rain yesterday 2 = DR = Rain today, dry yesterday 3 = DD = Dry today and dry yesterday with transition probability matrix .8 .2 0 0 0 0 .3 .7 P= .4 .6 0 0 0 0 .2 .8 Chapter 9 139 The equations for the limiting probabilities are: ∏0 = .8∏0 + .4∏2 ∏1 = .2∏0 + .6∏2 ∏2 = .3∏1 + .2∏3 ∏3 = .7∏1 + .8∏3 ∏0 + ∏1 + ∏2 + ∏3 = 1 which gives ∏0 = 4/15, ∏1 = ∏2 = 2/15, ∏3 = 7/15. ⎛ 5 ⎞15 ⎞ ⎢ ⎢⎝ 2 ⎠⎝ 2 ⎠= 70 323 ⎛ 20 ⎞⎢ ⎥⎝4⎠34. [∑ X] = ∑ E[X] = 2(n − 1)(1 − p) n E[number of changeovers] = E i i i =1 12. There are⎢ ⎢in⎝ 3 ⎠

3⎞ 2 ⎞⎛ 3⎞⎛ 3 ⎞⎛ 2 ⎞⎛ 3 ⎞⎛ 3⎞⎛ 7 ⎞⎛ 5⎞which neither feuding party serves; ⎢ ⎥⎢ ⎥in which the feuding women serves; and ⎝ 2 ⎠⎝ 3⎠⎛7⎞⎛5⎞ ⎥ ⎥ ⎥in which the feuding man serves. Since .95 = P{Z ≥ −1.64} it follows that p ≥ .95 if 2000 − 100n 30 n ≤ −1.64 or, equivalently, (2000 − 100n)/ n ≤ −49.2 and this will be the case if n ≥ 23. Now, with n E[Xj] =
P{Xj = 1} = ∏ (1 − 1/ i) , we have that i= j Var(Xj) = E[Xj](1 − E[Xj]). Then, because X is distributed as the conditional distribution of S given that S > 0, P{X = 1} = P{S = 1⎜S > 0} = λe − λ P{S = 1} ≈ P{S > 0} 1 − e − λ 21. Integrating the density function yields that that distribution function is e2 x / 2 , x>0 −2s 1 − e / 2, x > 0 which yields that
the inverse function is given by F(x) = F−1(u) = log(2u) / 2 if u < 12 − log(2[1 − u]) / 2 if u > 1 / 2 Hence, we can simulate X from F by simulating a random number U and setting X = F −1(U). P(M) + P(W) + P(G) − P(MW) − P(MG) − P(WG) + P(MWG) = .312 + .470 + .525 − .086 − .042 − .147 + .025 = 1.057 ⎛13 ⎞⎛ 52 ⎞(a) 4⎢ ⎥⎢ ⎥⎝5⎠⎝5⎠⎛ 4 ⎞⎛12 ⎞⎛
4 ⎞⎛ 4 ⎞⎛ 4 ⎞⎛ 52 ⎞(b) 13⎢ ⎥⎢ ⎥⎢ ⎥⎢ ⎥⎢ ⎥⎝ 2⎠⎝ 3 ⎠⎝ 1⎠⎝ 1⎠⎝ 1⎠⎝ 5 ⎠⎛13 ⎞⎛ 4 ⎞⎛ 4 ⎞⎛ 44 ⎞⎛ 52 ⎞(c) ⎢ ⎥⎢ ⎥⎢ ⎥⎢ ⎥⎝ 2⎠⎝ 2⎠⎝ 2⎠⎝ 1⎠⎝ 5 ⎠⎛ 4 ⎞⎛12 ⎞⎛ 4 ⎞⎛ 4 ⎞⎛ 52 ⎞(d) 13⎢ ⎥⎢ ⎥⎢ ⎥⎢ ⎥⎝3 ⎠⎝ 2 ⎠⎝1⎠⎝1⎠⎝ 5 ⎠⎛ 4 ⎞⎛ 48 ⎞⎛ 52 ⎞(e) 13⎢ ⎥⎢ ⎥⎢ ⎥⎝ 4⎠⎝ 1 ⎠⎝ 5 ⎠6 ·5· 4·3· 2 (a) 65 ⎛5⎞6 · 5 · 4⎢ ⎥⎝ 3 ⎠(d) 21 (g) 17. Let A be the event that you are dealt blackjack and let B be the event
that the dealer is dealt blackjack. Y ⎞4! ⎛ f X (1) , X (4) (x, y) = 2 x⎢ 2 zdz ⎥2 y , x < y ⎥2! ⎢⎝ X ⎠∫ = 48xy(y2 − x2). n −1 1 ns n −1 = n). e−3s/60 3. If U ≤ 1/2 then X = 6U − 3, whereas if U ≥ 1/2 then X is obtained by solving the quadratic 1/2 + X 2/32 = U in the region 0 ≤ X ≤ 4. (i) Say that trial i is a success if the ith pair selected have the same
number. Using the representation of Example 2l the variance can be n+m n+m computed by using From Example 8l ⎡0 ⎤ E[I1Il+j] = ⎨ n m n −1 ⎢⎣ n + m n + m − 1 n + m − 2 , j =1 , n −1 ≤ j < 1 ⎡0 ⎤ mn(m − 1)(n − 1) E[IiIi+j] = ⎨ ⎢⎣ (n + m)(n + m − 1)(n + m − 2)(n + m − 3) 45. e−tiM(t) = (pet + q)ne−ti and differentiation shows that the value of t
that minimizes it is such that iq npet = i(pet + q) or et = (n − i) p Using this value of t, the Chernoff bound gives that n ⎛iq ⎞+ q ⎞(n − i)i p i /(iq)i P{X ≥ i} ≤⎢ − n i ⎝⎠n i i (nq) (n − i) p = i i q i (n − i) n 12. a ⎝ a ⎠30. P(AB⎜A) = 2. Then E[Xj] = P{ball i is not in urn j, i ≥ j} = n ∏ (1 − 1/ i) i= j Hence, n (a) E[number of empty urns] = n ∑∑ (1 − 1/ i) j
=1 i = j (b) P{none are empty} = P{ball j is in urn j, for all j} n = ∏1/ j j =1 10. Hence, 2 4 E[X] = ∫x e 2 −x dx = Γ(3) = 2 0 9. EF = {(1, 2), (1, 4), (1, 6), (2, 1), (4, 1), (6, 1)}. 1 − e−1.4 − 1.4e−1.4 62. That is, the policyholder is more likely to me male if pm > pf (or more likely to be female if the inequality is reversed) than without this information,
thus raising the probability of a claim in the following year. x1 + d 0 1− d 1 ∫ ∫ dx dx n −1...dx2 dx1 x n − 3 + d x n − 2 + d x n −1 + d = [1 − (n − 1)d]n. ∑ 110 Chapter 7 44. 70 Chapter 5 38. 50. ⎛10 ⎞⎛5⎞5 4 1⎢ ⎥(1 / 3) (2 / 3) + (1/3) = 11/243 ⎝ 4⎠40. i 1 r −1 ⎞ ⎠⎛r⎞There are⎢ ⎥choices of the k of the x's to equal 0. 22 Chapter 3 19. Now after his
return it is equally likely that one door will have i and the other 5 − i pairs as it is that one will have i − 1 ant the other 6 − i. The maximization is similar. E ∪ F occurs if the sum is odd or if at least one of the dice lands on 1. Hence, E[X] = 8(12/47) = 96/47 Because E[X] > 2 it follows that there is at least one possible set of 12 consecutive components
that contain at least 3 failures. (a) 1 − e−3.5 − 3.5e−3.5 = 1 − 4.5e−3.5 (b) 4.5e−3.5 Since each flight has a small probability of crashing it seems reasonable to suppose that the number of crashes is approximately Poisson distributed. P{rejected} = 1 − (.9)4 56 Chapter 4 Theoretical Exercises 1. ∞ 3. 54. If it lands on heads then A will win with
probability Pn−1,m whereas if it lands tails then B will win with probability Pm,n (and so A will win with probability 1 − Pm,n). (b) Because Y + Z and Y are both linear combinations of the independent normal random variables Y and Z, it follows that Y + Z, Y has a bivariate normal distribution. The possibilities for part (b) become more numerous. ⎤Ij⎥
= ⎦n ∑ 1 n ∑P j independence not needed 1 ⎞I j ⎥⎥= ⎠n ∑ p (1 − p) j j independence needed 1 Let ⎧1 record at j Ij = ⎨ ⎩0 otherwise⎡ E⎢ ⎣n ∑ 1 Ij⎥ = ⎦⎛ Var ⎢⎢ ⎝124 n ∑ 1 n ∑ E[I j] = 1⎞I j ⎥⎥= ⎠n ∑ P{ X j is largest of X 1 ,..., X j } = 1 n ∑ 1 Var (I j) = n ∑1/ j 1 n 1⎛ 1⎞∑ j⎢⎝1 − j ⎥⎠1 Chapter 7 15. Since ∩ Ea is the empty set, it must have probability
0. Solving, gives P(A) = 36 p2 1 − 2 p (1 − p) Chapter 3 79. (a) P{N ≥ i} = 1 1 0 0 ∫ P{N ≥ i p}dp = ∫ (1 − p) i −1 (b) P{N = i} = P{N ≥ i} − P{N ≥ i + 1} = ∞ (c) E[N] = ∑ P{N ≥ i} = i=1 73. ⎛ ⎞Cov ⎢ X i , Y j ⎥= ⎢ i ⎥j ⎝⎠Now, E[Xi] = E[Yj] = 12/30 ∑ ∑ ∑∑ Cov(X ,Y) i i j j ⎛ 28 ⎞⎛ 30 ⎞E[XiYj] = P{red ball i and blue ball j are selected} =⎢ ⎥⎢ ⎥⎝10 ⎠⎝12
⎠Thus, ⎡⎛ 28 ⎞⎛ 30 ⎞⎤Cov(X, Y) = 80⎢⎢ ⎥⎢ ⎥− (12 / 30) 2 ⎥= −96/145⎣⎝10 ⎠⎝12 ⎠⎦116 Chapter 7 (b) E[XY⎜X] = XE[Y⎜X] = X(12 − X)8/20 where the above follows since given X, there are 12-X additional balls to be selected from among 8 blue and 12 non-blue balls. P{tails⎜w} = 3 1 5 1 36 + 75 111 + 15 2 12 2 39. Write Xn = ∑Z i where Zi is the
number of offspring of the ith individual of the (n − 1)st i =1 generation. Hence, the probability that the non-albino child is not a carrier is P(A, A⎜A, a or a, A or A, A) = 1 3 Where the first gene member in each gene pair is from the mother and the second from the father. For instance, suppose there are 10 cars, 9 transporting a single worker and the
other carrying 9 workers. Then E[max(Y1, Y2)] = E[Y1] + E[Y2] − E[min(Y1, Y2)] giving that E[min(Y1, Y2)] = 12 + 4 − 30. (a) number of nonnegative solutions of x1 + ... + x6 = 8 ⎛13 ⎞answer =⎢ ⎥⎝5⎠(b) (number of solutions of x1 + ... + x6 = 5) × (number of solutions of x1 + ... + x6 = 3) = ⎛10 ⎞⎛ 8 ⎞⎢ ⎥ ⎢ ⎥⎝ 5 ⎠⎝ 5⎠33. ⎛n⎞(d) there are⎢ ⎥k 3
selections in which the committee is of size k. Since E[X] = 50.4, Var(X) = 37.6992, E[Y] = 47.2, Var(Y) = 36.0608 it follows from the normal approximation to the binomial that is approximately distributed as a normal random variable with mean 50.4 and variance 37.6992, and that Y is approximately distributed as a normal random variable with
mean 47.2 and variance 36.0608. 20. Then E[Xi] = P{i − 1 is H, i is T} + P{i − 1 is T, i is H} = 2(1 − p)p, i ≥ 2. 52 ⎛ ⎞⎢ ⎥⎝13, 13, 13, 13 ⎠27. Hence, the Chernoff bound gives P{X ≤ i} ≤ ei−λ(λ/i)i 11. 12!/(12)12 48. P{A = w ⎜ 2w} = P{ A = w, 2 w} P{2w} P{ A = w, B = w, C ≠ w} + P{ A = w, B ≠ w, C = w} = P{2 w} 123 111 + 7 3 3 4 334 = = 1 2 3
1 1 1 2 2 1 11 + + 234 334 334 10. Condition on the initial flip. 4! 3! paths from A to the circled point; and paths from the circled point to B. (a) Since any guess will be correct with probability 1/n it follows that n E[N] = ∑ E[I] = n / n = 1 i i =1 (b) The best strategy in this case is to always guess a card which has not yet appeared. 95, centered eq.
509, lines 6 and 7: 73. 12 − i 66 ⎛ 12 − j ⎞⎛ 12 − i ⎞⎢⎥P{Y2 = i} = ⎢ ⎥⎢ ⎥ j ≠ i ⎢ 66 ⎝ 54 + j ⎠P{Y1 = i} = ∑ P{Y3 = i} = 11⎛ 12 − j ⎞⎛ 12 − k ⎞⎢⎥ ⎥⎢ 66 ⎝ 54 + j ⎠42 + k + j ≠i ∑∑⎢⎝ k≠ j ⎝ k ≠i ⎠⎢ j ⎥⎠All sums go from 1 to 11, except for prohibited values. It then puts element k + 1 in position k + 1; randomly chooses one of the positions 1, ..., k + 1 and
interchanges the element in that position with element k + 1. fY(x) = 1 fX((x − b)/a) if a > 0. For both roots to be real the discriminant (4Y)2 − 44(Y + 2) must be ≥ 0. Use the fact that a gamma (n, 1) random variable is the sum of n independent exponentials with rate 1 and thus has mean and variance equal to n, to obtain: { { ⎛ X −n ⎞ > .01⎬ = P X −
n / n > .01 n P⎨ ⎝ n ⎠ ≈ P N (0,1) > .01 n { = 2 P N (0,1) > .01 } } n} Now P{N(0, 1) > 2.58} = .005 and so n = (258)2. 1 94 Chapter 6 24. (b) If x < A then the strategy will accept the first value seen: if x > B then it will reject the first one seen; and if x lies between A and B then it will always yield return B. Now E[XY] = E[XE[X⎜Z]] = E[E[XE[X⎜Z]
⎜Z]] = E[E 2[X⎜Z]] = E[Y 2] X n −1 44. (a) P{X > 20} = e−1 (b) P{X > 30⎜X > 10} = 35. By Proposition 6.1 there are ⎛ n − 1⎞⎛ m − 1⎞2⎢ ⎥ ⎥number of outcomes and so ⎝ k − 1⎠⎝ k − 1 ⎠⎛ n − 1⎞⎛ m − 1⎞⎛ m + n ⎞P{2k runs} = 2⎢ ⎥ ⎢ ⎥ ⎣ 64. Method (b) is correct as it will enable one to estimate the average number of workers per car. (a) S = {(r, r), (r, g),
(r, b), (g, r), (g, g), (g, b), (b, r), b, g), (b, b)} (b) S = {(r, g), (r, b), (g, r), (g, b), (b, r), (b, g)} 2. Must show that E[Y 2] = E[XY]. There were 1 · 2 · 9 = 18 that started with a 4. Use the notation of the hint. Also let Z be a standard normal random variable. P(Ai) = (1/2)i, if i < n = (1/2)n−1, if i = n (a) (b) ∑ n i i =1 i (1 / 2) n + n(1 / 2) n −1 = 2 − 1 1 2 n −1
(c) Condition on whether they initially play each other. 0 ≤α ≤1 9. c. This occurs when P{X1 = X2 = X3} = 1. Hence, using this we obtain E[etX+sY⎜Y] = esYE[EtX⎜Y] = e sY eYt + t 2 /2 and so E[etX+sY] = et 2 /2 E[e(s + t)Y] = et 2 /2 (1 − s − t) −1 , s + t < 1 Setting first s and then t equal to 0 gives 2 E[etX] = et / 2 (1 − t) −1 , t < 1 E[esY] = (1 −
s)−1, s < 1 78. Assuming independence of the events that the⎢ ⎥ ⎥i 15 i = 3⎝ ⎠ 3 ⎠⎝ 3 ⎠devices exceed 15 hours. SD(aX + b) = 8. 3!23 25. ⎛ 52 ⎞⎢ ⎥⎝5⎠15. ⎛8⎞⎛ 2⎞⎛ 4⎞⎢ ⎥ ⎥ ⎥that ⎝ 3⎠⎝ 1 ⎠⎝ 2 ⎠⎝ 6⎞⎛ 6⎞⎛ 2⎞⎛ 6⎞⎛ 6⎞(b) There are⎢ ⎥ ⎥⎢ ⎥+⎢ ⎥ ⎥ ⎥= 1000 possible committees. 29. Hence, there are 5!⎢ ⎥⎢ ⎝ 5 ⎠⎝ 5 ⎠possible results. ⎛ M ⎞⎛ N ⎞⎢ ⎥ ⎥ ⎝ k ⎠⎝ r − k ⎠
⎝M + N ⎞⎢ ⎝ r ⎠16. (a) E[X] = E[X⎜type 1]p + E[X⎜type 2](1 − p) = pμ1 + (1 − p)μ2 (b) Let I be the type. Let X be uniform on (0, 1) and define Ea to be the event that X is unequal to a. Pa(F) = 1 ni ∑ P(F S) P(S) = ∑ i m i i i Pb(F) = i m i ini ∑ Thus, we must show that ∑ i n ∑ n / i ≥ m i i i 2 i or, equivalently, ∑ i n ∑ n i i j /j≥ j ∑n ∑n i i j j or, i ∑∑ j n n ≥
∑∑ n n i i≠ j j i j i≠ j Considering the coefficients of the term ninj, shows that it is sufficient to establish that i j + j ≥2 j i or equivalently i2 + j2 ≥ 2ij which follows since (i − j)2 ≥ 0. Given this choice the other r − k of the ⎝k ⎠x's must be positive and sum to n. The equations for the limiting probabilities are: ∏c = .7∏c + .4∏s + .2∏g ∏s = .2∏x + .3∏s +
.4∏g ∏g = .1∏c + .3∏s + .4∏g ∏c + ∏s + ∏g = 1 and the solution is: ∏c = 30/59, ∏s = 16/59, ∏g = 13/59. 1000 II 7000 19000 (a) (b) (c) (d) (e) 20,000 12,000 11,000 68,000 10,000 1000 3000 0 III Chapter 2 11 14. Also, for i ≠ j ≠ k ≠ r, P(Ai,jAk,r) = 1/(365)2. P{r successes before m failures} = P{rth success occurs before trial m + r} m + r − 1
⎛ n − 1⎞r n−r = ⎢ ⎥p (1 − p) . αp 1− p + ∞ ∑αp (1/ 2) n n n =1 (a) If X is binomial (n, p) then, from exercise 15, P{X is even} = [1 + (1 − 2p)n]/2 = [1 + (1 − 2λ/n)n]/2 when λ = np → (1 + e−2λ)/2 as n approaches infinity (b) P{X is even} = e−λ 18. (a) P(A) = P12 + 1 − P12 1 − P22 P(A) or P(A) = ( )([ )] P12 P12 + P22 − P12 P22 (c) similar to (a)
with Pi 3 replacing Pi 2 . I= ∫∫∫∫∫ f(x1) … f(x5)dx1...dx5 x1 < x2 > x3 < x4 > x5 ∫∫∫∫∫ du … du5 u1 < u2 > u3 < u4 > u5 1 0 < ui < 1 = ∫ ∫ ∫ ∫ u2 du2 ...du5 = by ui = F(xi), i = 1, ..., 5 = ∫ ∫ ∫ (1 − u32) / 2 du3... = ∫ ∫[u4 − u43 / 3] / 2du4 du5 1 ∫ = [u 2 − u 4 / 3] / 2du5 = 2/15 0 12. (a) p(0, 0) = (8/13)2, p(0, 1) = p(1, 0) = (5/13)(8/13), p(1, 1) = (5/13)2
(b) p(0, 0, 0) = (8/13)3 p(i, j, k) = (8/13)2(5/13) if i + j + k = 1 p(i, j, k) = (8/13)(5/13)2 if i + j + k = 2 5. (a) P(M) = P(M⎜A)P(A) + P(M⎜R)P(R) = (.15)(.6) + (.05)(.4) = .11 Chapter 3 29 (b) P(T⎜Mc) = = P (T) P(M c) P(T A) P(A) + P (T R) P(R) 1 − P(M) (.2)(.6) + (.1)(.4) 16 = = .89 89 c c c (c) P(A⎜M T W) = = (d) P(A⎜Th) = P(M cT cW c A) P(A) P(M
cT cW c) (1 − .15 − .20 − .25)(.6) 12 = (.4)(.6) + (.75)(.4) 27 P(Th A) P(A) P (Th) (.15)(.6) 3 = = (.15)(.6) + (.15)(.4) 5 (e) P(A⎜no mail) = P(no mail A) P (A) P(no mail) (.15)(.6) 9 = = (.15)(.6) + (.4)(.4) 25 53. 25. The above following as the left hand event is the union of the 4 mutually exclusive right hand events. By induction: (x1 + x2 + ... + xr)n = n
⎛n⎞i1 = 0 1 ∑⎢⎝ i ⎠x + ... dx = 2 x 20 x 20 ∫ ∫ y (b) F(y) = 10 ∫x 2 dx = 1 − 10 i 6 (c) 5. E[cX] = cp + c−1(1 − p) Hence, 1 = E[cX] if cp + c−1(1 − p) = 1 or, equivalently pc2 − c + 1 − p = 0 or (pc − 1 + p)(c− 1) = 0 Thus, c = (1 − p)/p. ⎛10 ⎞⎛12 ⎞There are⎢ ⎥⎢ ⎥possible choices of the 5 men and 5 women. 3 ⎡⎤ 25 + 1170 ⎤⎥ Hence P ⎨Y > ⎬≤ .1. 19.
When α > 0 x−β⎤ ⎛x−β ⎞ ⎡ P{αX + β ≤ x} = P ⎨ X ≤ ⎬ = F⎢ α⎥ ⎝ α ⎠ When α < 0 x−β⎤ ⎛x−β ⎞ ⎡ P{αX + β ≤ x} = P ⎨ X ≥ ⎬ = 1 − F⎢ α ⎥ . In addition, the conditional distribution of X given that Y = y is exponential with mean y. P(E3⎜E1E2) = ⎢ ⎥ ⎥⎝ 1 ⎠⎝ 12 ⎠⎝ 13 ⎠Hence, ⎛ 4 ⎞⎛ 48 ⎞⎛ 52 ⎞⎛ 3 ⎞⎛ 36 ⎞⎛ 39 ⎞⎛ 2 ⎞⎛ 24 ⎞⎛ 26 ⎞p =⎢ ⎥ ⎥ ⎥⎢ ⎥ ⎥ ⎥⎢ ⎥ ⎥ ⎥⎝ 1 ⎠⎝ 12 ⎠⎝
13 ⎠⎝ 1 ⎠⎝ 12 ⎠⎝ 13 ⎠⎝ 1 ⎠⎝ 12 ⎠⎝ 13 ⎠14. Then Yi is geometric with parameter 4⎡ 3 ⎤4 4 25 E[Y] = E⎢ Yi ⎥ = 1 + + + 4 = 3 2 3⎣ i=0 ⎦∑ ⎛ 3 ⎞4 130 Var[Y] = Var⎢⎢ Yi ⎥⎥ = + 2 + 12 = 9⎢ i =0 ⎠9 ∑ Chapter 8 135 Hence, ⎛ 25 25 1300 ⎞⎟ 1 P⎨ Y − > ⎬≤ 3 3 9 ⎟ 10 ⎢ and so we can take a = 25 − 1300 25 + 1300 , b= . If we let Xi equal 1 if all the elements
of Ai are similarly colored, and let it be 0 otherwise, then ∑ r i =1 X i is the number of subsets whose elements all have the same color. 1/2 9. There is 1 outcome when the second die lands on two, 2 when it lands on three, 3 when it lands on four, 4 when it lands on five, and 5 when it lands on six. 1 1 a E[X − Y] = ∫∫ x − y a dydx . 64 Chapter 5 ∞ 6. 4
9 23 24.⎡ 2 3 ⎤(a) 1 − F(2) = exp⎢ − t dt ⎥ = e−4⎣⎦ 0 ⎦∫ (b) exp(−(.4)4/4] − exp[−(1.4)4/4] ⎤⎡ 2 (c) exp⎢ − t 3dt ⎥ = e−15/4 ⎣⎦⎣ 1 ∫ 37. 15. ⎛ X − 40 10 ⎞ > ⎬ = 1 − Φ(2.5) = 1 − .9938 P{X > 50} = P ⎨ 4⎢ ⎣ 4 10 Hence, (P{X < 50}) = (.9938)10 E[Points] = 10(1/10) + 5(2/10) + 3(2/10) = 2.6 ⎛ X − 5 9 − 5 ⎞ .2 = P ⎨ > ⎬ = P{Z > 4/σ} where Z is a standard
normal. Now, consider that component along with the next 11 when moving in a clockwise manner and let X denote the number of failures in that group of 12. 1/n if discard, 46. pn = r1!...rn ! ⎛ n ⎞⎢ ri ⎥! ⎢⎝ 1 ⎠∑ ∑ n = ∏e λ i =1 3. g(x) = xn(n−1) is convex. The Jacobian of the transformation is J= 1 1/ y 0 − x / y2 = −x / y2 −1 Hence, J = y 2 / x . P{1}
= 4 1 = 44 64 84 ⎛ 4 ⎞⎛ 4 ⎞P{2} = ⎢ ⎥4 + ⎥ ⎥+ 4 ⎥44 = 256⎝ 2 ⎠⎝ 2 ⎠⎦⎝ 4 ⎠⎝ 3 ⎞4! 4 36 4 = P{3} = ⎢ ⎢64 ⎝ 3 ⎠⎝ 1 ⎠2! 4! 6 P{4} = 4 = 64 4 54 64 41. 8 ⎞r ∑ m + r − 1 ⎥. Each path is a linear arrangement of 4 r's and 3 u's (r for right and u for up). If we let Xi denote the life of bulb i and let Ri be the time to replace bulb i then the desired 99 ⎡ 100 ⎤
probability is P ⎨ X i + Ri ≤ 550⎬ . ⎡10, 000 ⎤ P⎨ X i > 2,700,000⎬ ≈ P{Z ≥ (2,700,000 − 2,400,000)/(800 · 100)} = P{Z ≥ 3.75} ≈ 0 ⎣ i =1 ⎦ ∑ Let Yi denote the additional number of fish that need to be caught to obtain a new type when 4−i . ⎛n⎞⎢ ⎥⎝k ⎠(n − 1)! (n − 1)! ⎛ n − 1⎞⎛ n − 1⎞+⎢ ⎥+⎢ ⎥= r !(n − 1 − r)! (n − r)!(r − 1)! ⎝ r ⎠⎝ r − 1 ⎠n! ⎡n − r r ⎤
⎝n⎠= + ⎥=⎢ ⎥r !(n − r)! ⎢⎣ n⎦ n⎠⎝ r ⎠⎝ n + m ⎠⎝ n ⎞⎝ m ⎞There are⎢ ⎥gropus of size r. (a) x1 + x2 + x3 + x4 = 20, x1 ≥ 2, x2 ≥ 2, x3 ≥ 3, x4 ≥ 4 Let y1 = x1 − 1, y2 = x2 − 1, y3 = x3 − 2, y4 = x4 − 3 y1 + y2 + y3 + y4 = 13, y1 > 0 ⎛12 ⎞Hence, there are⎢ ⎥ = 220 possible strategies. ⎡ ⎤ L− X ⎞⎛ X , P ⎨min⎢ ⎥< 1 / 4⎬ X ⎠⎝L− X ⎠⎝ ⎤ L− X ⎞⎛ X , = 1 − P
⎨min⎢ ⎥> 1 / 4⎬ X ⎠⎝L− X ⎞⎝ L− X ⎞ ⎡ X = 1 − P⎨ > 1 / 4, > 1 / 4⎬ L − X X ⎦ = 1 − P{X > L/5, X < 4L/5} ⎡L ⎤ = 1 − P⎨ < X < 4 L / 5⎬ ⎣5 ⎦ 3 2 =1− = . 1 ⎤ 1 exp{− (u + v + w)} , u + v > w, u + w > v, v + w + u 2 ⎦ 2 P(Yj = ij, j = 1, ..., k + 1} = P{Yj = ij, j = 1, ..., k} P(Yk+1 = ik+1⎜Yj = ij, j = 1, ..., k} k k !(n − k)! = P{n + 1 − Yi = ik +1 Y j = i j , j
= 1,..., k} n! i =1 ∑ k!(n − k)!/n!, if k +1 ∑i j = n +1 j =1 = 0, otherwise Thus, the joint mass function is symmetric, which proves the result. Hence, E[NiNj] = p ([n nE[N] − E [N]) = 1− p 1− p pj i i 2 i j 2 pi − n 2 pi2 − npi (1 − pi)] i = n(n − 1)pi pj and Cov(Ni, Nj) = n(n − 1)pi pj − n2pi pj = −npi pj Chapter 7 127 31. E[N(t)⎜N(t − 1)] = N(t − 1) −
E[number selected] r = N(t − 1) − N(t − 1) b+ w+r b+w E[N(t)⎜N(t − 1)] = N(t − 1) b+ w+r t ⎛ b+w ⎞E[N(t)] = ⎢ ⎥w ⎝b+ w+r ⎠32. (a) E[time of first type 1 catch] − 1 = 1 − 1 using the formula for the mean of a geometric p1 random variable. (a) 1 − e−3 − 3e−3 − e−3 32 17 = 1 − e−3 2 2 17 1 − e−3 P{ X ≥ 3} 2 = (b) P{X ≥ 3⎜X ≥ 1} = P{ X ≥ 1}
1 − e−3 59. (a) 6! (b) 3!2!3! (c) 3!4! 12. (b) Let A be the event that A wins. r +b+ g r +b+ g r +b bg b g Hence, the answer is . (a) E[Tr⎜Tr−1] = Tr−1 + 1 + (1 − p)E[Tr] (b) Taking expectations of both sides of (a) gives E[Tr] = E[Tr−1] + 1 + (1 − p)E[Tr] or E[Tr] = 1 1 + E[Tr −1] p p (c) Using the result of part (b) gives E[Tr] = 1 1 + E[Tr −1] p p ⎞1
1/1 1 + ⎥ + E[Tr − 2] ⎥p p⎝ p p ⎠2 2 = 1/p + (1/p) + (1/p) E[Tr−2] = 1/p + (1/p)2 + (1/p)3E[Tr−3] = r = ∑ (1/ p) i ∑ (1/ p) i i =1 r = (1 / p) r E[T0] since E[T0] = 0. ⎛ 5⎞⎛ 6⎞⎛ 4⎞⎢ ⎥ ⎥ ⎥= 600 ⎝ 2 ⎠⎝ 2 ⎠⎝3 ⎠⎛8⎞⎛ 4⎞⎛8⎞⎛ 2⎞⎛ 4⎞(a) There are⎢ ⎥ ⎥⎢ ⎥+⎢ ⎥ ⎥ ⎥= 896 possible committees. ⎡ X − 74 ⎤ (a) P{ X > 80} = P⎨ > 15 / 7 ⎬ ≈ PPZ > 2.14} ≈ .0162
⎝ 14 / 5 ⎠ ⎡ Y − 74 ⎤ > 24 / 7 ⎬ ≈ P{Z > 3.43} ≈ .0003 (b) P{Y > 80} = P ⎨ ⎝ 14 / 8 ⎠ (c) Using that SD (Y − X) = 196 / 64 + 196 / 25 ≈ 3.30 we have P{Y − X > 2.2} = P{Y − X } / 3.30 > 2.2 / 3.30} ≈ P{Z > .67} ≈ .2514 (d) same as in (c) 14. (a) P(S ∪ F ∪ G) = (28 + 26 + 16 − 12 − 4 − 6 + 2)/100 = 1/2 The desired probability is 1 − 1/2 = 1/2. 1 −1 / 2
e 2 P{beneficial⎜2} = P{2 beneficial}3/4 P{2 beneficial}3 / 4 + P{2 not beneficial}1 / 4 e−3 = e−3 32 3 2 4 32 3 −5 52 1 +e 2 4 2 4 61. Because the non-albino child has an albino sibling we know that both its parents are carriers. ψ(t) = log φ(t) ψ ′(t) = φ′(t)/φ(t) ψ ″(t) = ψ ″(t) Chapter 7 φ (t)φ ″(t) − (φ ′(t))2 φ 2 (t) t =0 = E[X 2] − (E[X])2 =
Var(X). Easiest to first take log and then determine the p that maximizes log P{X = k}. Let X denote the number in favor. 1 (1) 4 4 3 P{2 headed⎜heads} = = = . Hence, by the induction hypothesis ⎛⎛ s − t ⎞n −1 ⎢ ⎥P{X(k) − X(k−1) > t⎜X(n) = s} = ⎨⎝ s ⎠ ⎩0 and thus, for t < a, ⎛ s−t ⎞P{X(k) − X(k−1) > t = ⎢ ⎥s ⎠t ⎝ ∫ n −1 if t < s if t > s ns n −1
(a−t ⎞ds = ⎢ ⎥n a ⎝ a ⎠n ⎝s⎠which completes the induction. Therefore, E[X 2] = E[X 2⎜G]P(G) + E[X 2⎜B]P(B) = 12(.4) + 30(.6) = 22.8 Consequently, Var(X) = 22.8 − (4.2)2 = 5.16 66. Condition on the first ball selected to obtain Ma,b = b a M a ,b −1 , a, b > 0 M a −1,b + a+b a+b Ma,0 = a, M2,1 = 37. (a) P(F⎜C) = P(FC) = .02/.05 = .40 P (C) (b)
P(C⎜F) = P(FC)/P(F) = .02/.52 = 1/26 ≈ .038 21. (b) From (a) E[Cov(X, Y⎜Z)] = E[XY] − E[E[X⎜Z]E[Y⎜Z]] On the other hand, Cov(E[X⎜Z], E[Y⎜Z] = E[E[X⎜Z]E[Y⎜Z]] − E[X]E[Y] and so E[Cov(X, Y⎜Z)] + Cov(E[X⎜Z], E[Y⎜Z]) = E[XY] − E[X]E[Y] = Cov(X, Y) (c) Noting that Cov(X, X⎜Z) = Var(X⎜Z) we obtain upon setting Y = Z that Var(X) = E[Var(X⎜Z)] +
Var(E[X⎜Z]) 21. n j =0 88. n Then each Xi is an identically distributed geometric random variable and ∑X i , representing i =1 the number of trials needed to amass n successes, is a negative binomial random variable. 6·5·4 5 = 6·6·6 9 1 1 = 3! 6 51 5 = 9 6 54 P(w⎜w transferred}P{w tr.} + P(w⎜R tr.}P{R tr.} = P{w transferred⎜ w} = Chapter 3 P{w
w tr.}P{w tr.} P{w} 21 12 4 + = . ⎛n⎞n−m n⎢ ⎥(n − 1) / N m ⎝ ⎠52. Then, Di = k ≠ i X i , k , and so, for i ≠ j ∑ ⎛ ⎞Cov(Di, Dj) = Cov ⎢ X i , k , X r , j ⎥ k ≠i ⎥r≠ j ⎝ ⎠∑ = ∑∑ Cov(X ∑ i,k , X r , j) k ≠i r ≠ j = Cov(Xi, j , Xi, j) = Var(Xi, j) = p(1 − p) where the third equality uses the fact that except when k = j and r = i, Xi ,k and Xr, j are independent and
thus have covariance equal to 0. y 6 −i 10 ⎛ 6 ⎞⎛ 2 ⎞⎛ 1 ⎞since F (15) = . The answer is 5/12, which can be seen as follows: 1 = P{first higher} + P{second higher} + p{same} = 2P{second higher} + p{same} = 2P{second higher} + 1/6 Another way of solving is to list all the outcomes for which the second is higher. P{no acc.} 7 3 (.4)(.6) + (.2)(.8) 46
10 . P(EFc ∪ EcF) = P(EFc) + P(EcF) = P(E) − P(EF) + P(F) − P(EF) 13. p(1 − p)i −1 p(1 − p) n − i −1 1 = n −1 ⎛ n − 1⎞p 2 (1 − p) n − 2⎢ 1 ⎥⎝ ⎠P{ X = k , X + Y = m} P{ X + Y = m} P{ X = k , Y = m − k} = P{ X + Y = m} P{X = k⎜X + Y = m} = ⎛ n ⎞p k (1 − p) n − k ⎛ n ⎞p m − k (1 − p) n − m + k⎢k ⎥m − k ⎥⎝ ⎠= ⎝ ⎠2 n ⎛ ⎞p m (1 − p) 2 n − m
⎢m ⎥⎝ ⎠⎝ n ⎠⎝ k ⎠⎢ m − k ⎥⎝ ⎠= ⎝ ⎠⎝ 2 n ⎛ ⎞⎝m ⎥⎝ ⎠16. (iii) n(n − 1)2n − 2 (c) From a set of n we want to choose a committee, its chairperson its secretary and its treasurer (possibly the same). That is, there must be at least 365 log(2) people. Then ⎛ 4 ⎞P{N = 5} = 2⎢ ⎥(1 / 2)(1 / 2) 4 = 1/4 ⎝1⎠P{N = 4} = 2(1/2)4 = 1/8, ⎛5⎞P{N = 7} = 5/16 P{N = 6} = 2⎢
⎝(1 / 2) 2 (1 / 2) 4 = 5/16, ⎝ 2⎠E[N] = 4/8 + 5/4 + 30/16 + 35.16 = 93/16 = 5.8125 74. (a) 1/16, 74. 47. (a) P(2 arrivals in (0, s) ⎜2 arrivals in (0, 1)} =P{2 in (0, s), 0 in (s, 1)}/e−λλ2/2) = [e−λs(λs)2/2][e−(1−s)λ]/(e−λλ2/2) = s2 = 1/9 when s = 1/3 (b) 1 − P{both in last 40 minutes)} = 1 − (2/3)2 = 5/9 2. 2: change “when j > r.” to “when j < 0.” p.
Another argument is to choose a child at random. Hence, there are 10 · 9 · 8 · · · 5 · 4 = 604,800 possibilities. 52. Write X = σZ + μ where Z is a standard normal random variable. Then, with p = P(C), P(C⎜E) = P (E C) P (C) P(E C) P (C) + P (E C c) P(C c) Similarly, P(C⎜Ec) = = 28 P (E c C) P (C) P(E c C) P (C) + P (E c C c) P(C c) .732 p
.732 p + .865(1 − p) Chapter 3 50. dn When n = 2j the nth derivative will equal n ⎨t n ⎬ /(j !2 j) plus constants multiplied by powers dt of t. Suppose the present state is i, where i > 0. Chapter 7 101 15. P{new} = ∑ P{new i 30 type i}pi = ∑ (1 − p) i n −1 pi i Chapter 3 57. From this it is easy to see that the transition matrix Pij, i, j = 0, 1, ..., 5 is as
follows: 1/ 2 0 1/ 4 1/ 4 P= 0 0 0 0 0 1/ 2 1/ 4 1/ 4 0 1/ 4 1/ 4 1/ 4 1/ 4 0 0 0 0 1/ 2 1/ 2 0 1/ 4 1/ 4 1/ 4 1/ 4 0 0 1/ 4 1/ 4 0 0 1/ 4 1/ 4 Since this chain is doubly stochastic (the column sums as well as the row sums all equal to one) it follows that ∏i = 1/6, i = 0, ..., 5, and thus he runs barefooted one-sixth of the time. Then E[X i)] = 8, i = 1,2 E{ X i] = 8 /
3, i = 3,4 E[min(X 1 , X 2)] = 4 E[min(X i , X j)] = 2, i = 1,2, j = 3,4 E[min(X 3 , X 4)] = 4 / 3 E[min(X 1 , X 2 , X j)] = 8 / 5, j = 3,4 E[min(X i , X 3, X 4)] = 8 / 7, i = 1,2 E[min(X 1 , X 2 , X 3 , X 4] = 1 (a) E[max Xi] = 2 · 8 + 2 · 8/3 − (4 + 4 · 2 + 4/3) + (2 · 8/5 + 2 · 8/7) − 1 = 437 35 (b) E[max(X1, X2)] = 8 + 8 − 4 = 12 (c) E[max(X3, X4)] = 8/3 +
8/3 − 4/3 = 4 (d) Let Y1 = max(X1, X2), Y2 = max(X3, X4). There will be 2k runs if the outcome is either of the form y1, x1, ..., yk xk or x1y1, ... xk, yk where all xi, yi are positive, with x1 + ... + xk = n, y1 + ... + yk = m. ≥ n −1 ∑ P(E) − (n − 2) + P(E) − 1 by induction hypothesis n i 1 19. Note that at birth, Smith was equally likely to receive either a
blue gene or a brown gene from each parent. (a) The number of vectors that have xk = j is equal to the number of vectors x1 ≤ x2 ≤ ... ≤ xk−1 satisfying 1 ≤ xi ≤ j. ⎬ = 1 − lim+ F⎜ h → 0 α ⎠ ⎝ α ⎠ ⎝ ∞ 6. 0 16. 4/36 + 4/36 +1/36 + 1/36 = 5/18 20. ∑ E[I I bank rolls i]P{bank rolls i} = ∑ (P{roll is greater than i}) P{bank rolls i} E[I1I2] = 1 2 i =2 2 i = E[
I12] ≥ (E[I1])2 = E[I1] E[I2] 47. Also, P{X ≤ a1, Y ≤ b2} = P{X ≤ a1, b1 < Y ≤ b2 } + P{X ≤ a1, Y ≤ b1} and similarly, P{X ≤ a2, Y ≤ b1} = P{a1 ≤ X ≤ a2, < Y ≤ b1 } + P{X ≤ a1, Y ≤ b1}. The probability they will last for at least 2000 hours is ⎤ ⎡ n ⎡ 2000 − 100n ⎤ p = P ⎨ X i ≥ 2000⎬ ≈ P ⎨Z ≥ ⎬ 30 n ⎦ ⎣ ⎦ ⎣ i =1 ∑ where Z is a standard normal
random variable. 41. e−1 34. and the minimum when i is 2 b Var(X) = b b Follows since, given X, g(X) is a constant and so E[g(X)Y⎜X] = g(X)E[Y⎜X] 27. P(m counted) = ∑ P(m n events)e λ λ − n ∞ = ⎛n⎞∑⎢⎝ m ⎥⎠p m n / n! (1 − p) n − m e − λ λn / n ! n=m = e − λp (λp) m m! e − λp = ∞ [λ (1 − p)]n − m − λ (1− p) e (n − m)! n=m ∑ (λp) m m!
Intuitively, the Poisson λ random variable arises as the approximate number of successes in n (large) independent trials each having a small success probability α (and λ nα). (a) 11111 1 = 2 2 2 2 2 32 (b) 13131 9 = 2 4 2 4 2 128 (b) 11111 1 = 2 2 2 2 2 32 (c) 18 128 (c) 1 16 (d) 110 128 (d) 15 16 (a) P{carrier⎜3 without} 1/ 8 1/ 2 = 1/9. P(E⎜F) =
P(EF)/P(F) P(E⎜FG)P(G⎜F) = P(EFG) P (FG) P(EFG) = P (FG) P(F) P(F) P(E⎜FGc)P(Gc⎜F) = P(EFG c) . For ε > 0, let δ > 0 be such that ⎜g(x) − g(c) ⎜ < ε whenever ⎜x − c⎜ ≤ δ. ⎡ E⎢ ⎣ E[X i] + 5 ∑ Xi + 1 8 ∑ 1 ⎤Yi ⎥ = ⎦5 ∑ 1 =5 24. i 1 14. Assume it when n + m = k. (a) The probability that a family has 2 sons is 1/4; the probability that a family
has exactly 1 son is 1/2. P(X = n, Y = m) = ∑ P (X = n, Y = m X 2 i = e − (λ1 + λ2 + λ3) min(n , m) ∑ i =0 92 λ1n −i = i) P(X 2 = i) λ3m −i λ2i (n − 1)! (m − i)! i ! Chapter 6 17. Hence, the desired probability is e−1 = .368 which is not particularly close to the exact solution of .2656 provided in Example 5n of Chapter 2, thus indicating that n = 10
is not large enough for the approximation to be a good one. (a) Use all your money to buy 500 ounces of the commodity and then sell after one week. A choice of r elements from a set of n elements is equivalent to breaking these elements into two subsets, one of size r (equal to the elements selected) and the other of size n − r (equal to the elements
not selected). Hence, as there are⎢ ⎥such subsets. (a) (.9)(.8)(.7) = .504 (b) Let Fi denote the event that she failed the ith exam. ⎝ r − k − 1 ⎠⎝ n − r + k ⎠Hence the result follows. 64 Chapter 3 72. (b) f(x, y) = 1/4, −1 ≤ x, y ≤ 1 = f(x)f(y) where f(v) = 1/2, −1 ≤ v ≤ 1. Assume that Pa,b = 1/2 when k ≥ a + b and now suppose a+ b = k + 1. Follows from
the formula for the joint moment generating function. Let A denote the event that the next card is the ace of spades and let B be the event that it is the two of clubs. J= 1 1 ⎛ x 1 ⎞−1 − (v + 1) 2 ⎥ ( ) x y = + = = − + ⎢ y2 y ⎥y2 1/ y − x / y 2 u ⎠⎝ fu,v (u, v) = 57. P(L3 F1 S1) P(F1S1) P(L3) = (1/ 2)(1/ 9) = 1/ 6 1/ 3 (a) Because there will be 4 games if
each player wins one of the first two games and then one of them wins the next two, P(4 games) = 2p(1 − p)[p2 + (1 − p)2]. 4. 11 1 (a) P{fair⎜h} = 2 2 = . Cov(X 1 + X 2 , X 2 + X 3) (a) Var(X 1 + X 2) Var(X 2 + X 3) = , j =1 , n −1 ≤ j < 1 1 2 (b) 0 12 46. (a) E (b) EF (c) EG ∪ F 8. 8·7 = 14/39, 13 · 12 8·5 = 10/39 p(0, 1) = p(1, 0) = 13 · 12 5·4 p(1,
1) = = 5/39 13 · 12 (a) p(0, 0) = (b) p(0, 0, 0) = 8·7·6 = 28/143 13 · 12 · 11 8·7·5 = 70/429 13 · 12 · 11 8·5· 4 p(0, 1, 1) = p(1, 0, 1) = p(1, 1, 0) = = 40/429 13 · 12 · 11 5·4·3 p(1, 1, 1) = = 5/143 13 · 12 · 11 p(0, 0, 1) = p(0, 1, 0) = p(1, 0, 0) = 3. 1− 42. S 14 F 10 10 2 2 4 8 G (c) since 50 students are not taking any of the courses, the probability that
neither one is ⎛ 50 ⎞⎛100 ⎞taking a course is⎢ ⎥ ⎥= 49/198 and so the probability that at least one is taking a ⎝2⎠⎝ 2 ⎠course is 149/198. Using the representation N=2+I where I is 0 if the first two games are won by the same team and 1 otherwise, we have that Var(N) = Var(I) = E[I]2 − E2[I] Now, E[I]2 = E[I} = P{I = 1} = 2p{1 − p} and so Var(N)
= 2p(1 − p)[1 − 2p(1 − p)] = 8p3 − 4p4 − 6p2 + 2p Differentiation yields d Var(N) = 24p2 − 16p3 − 12p + 2 dp and it is easy to verify that this is equal to 0 when p = 1/2. c c Hence, cX is gamma with parameters (t, λ/c). p(0) = P{no sale on first and no sale on second} = (.7)(.4) = .28 p(500) = P{1 sale and it is for standard} = P{1 sale}/2 =
[P{sale, no sale} + P{no sale, sale}]/2 = [(.3)(.4) + (.7)(.6)]/2 = .27 p(1000) = P{2 standard sales} + P{1 sale for deluxe} = (.3)(.6)(1/4) + P{1 sale}/2 = .045 + .27 = .315 p(1500) = P{2 sales, one deluxe and one standard} = (.3)(.6)(1/2) = .09 p(2000) = P{2 sales, both deluxe} = (.3)(.6)(1/4) = .045 14. Hence, P{Mk = 0} ≈ exp[−k/(2n − 1)] (iii) and
(iv) P{T > αn} = P{Mαn = 0} ≈ exp[−αn/(2n − 1)] → e−α/2 60 Chapter 4 23. (a) P{.9000 − .005 < X < .9000 + .005} .005 ⎞ ⎡ .005 = P ⎨− ⎬ ⎢ 2,500 ⎦ = P{Z > 15.99} ≈ negligible. 309, first line of Example 8b: change “let Yi denote the selection ” to “let Yi denote the selection ” p. 83. Now, since X is a hypergeometric random variable it follows that
E[X] = 12(10/30) = 4 and E[X 2] = 12(18)(1/3)(2/3)/29 + 42 = 512/29 As E[Y] = 8(12/30) = 16/5, we obtain E[XY] = 2 (48 − 512 / 29) = 352/29, 5 and Cov(X, Y) = 352/29 − 4(16/5) = −96/145 64. 1 1 1 1 3 4+2+3 9 (1) + + 3 32 34 45. λ(t) = 16. (a) All we know when the procedure ends is that the two most flips were either H, T, or T, H. We have
assumed that the weekly sales are independent. 1 + 1/ 2 3 Let C be the event that the patient has cancer, and let E be the event that the test indicates an elevated PSA level. 416, line 1: change “Let X 1, . (a) P{⎜X⎜ > 1/2} = P{X > 1/2} + P{X < −1/2} = 1/2 (b) P{⎜X⎜ ≤ a} = P{−a ≤ X ≤ a} = a, 0 < a < 1. 13. 12! ⎛ 12 ⎞⎢ ⎥= ⎝ 3, 4, 5 ⎠3!4!5! 28. Hence,
the probability that neither 52 · 51 is dealt blackjack is .9017. P{C⎜woman} = P{women C}P{C} P{women A}P{ A} + P{women B}P{B} + P{women C}P{C} 100 1 225 = = 50 75 100 2 .5 + .6 + .7 225 225 225 .7 37. i =1 128 Chapter 7 35. Then P(T C1) P (C1) P(C1⎜T) = P(T C1) P (C1) + P (T C2) P (C2) 3(.4) 2 (.6) = .395 3(.4) 2 (.6) + 3(.7) 2
(.3) = Now, with Nj equal to the number of heads in the final j flips, we have E[N10⎜T] = 2 + E[N7⎜T] Conditioning on which coin is being used, gives E[N7⎜T] = E[N7⎜TC1]P(C1T) + E[N7TC2]P(C2⎜T) = 2.8(.395) + 4.9(.605) = 4.0705 Thus, E[N10⎜T] = 6.0705. dp = 1 / i 1 i (i + 1) ∞ ∑1/ i = ∞ . (a) P{no acc., acc. 30. 87. ⎝ k − 1⎠⎝ k − 1 ⎠⎝ n ⎠There will
be 2k + 1 runs if the outcome is either of the form x1, y1, ..., xk, yk, xk+1 or y1, x1, ..., yk, xk yk + 1 where all are positive and xi = n, yi = m. α ∑⎢ ⎥p1i (1 − p1)n − i + (1 − α)∑⎢ ⎥p2i (1 − p2) n − i 45. Pn,i = pnP−1,i +1 + (1 − p) Pn −1,i −1 20. ⎛3⎞⎛n⎞1/2 = ⎢ ⎥ ⎥or n(n − 1) = 12 or n = 4. FY(y) = P{log X ≤ y} = P{X ≤ ey} = FX(ey) fY(y) =
fX(ey)ey = e y e − e 40. Let Xi equal 1 if both choose item i and let it be 0 otherwise; let Yi equal 1 if neither A nor B chooses item i and let it be 0 otherwise. That is, the number of vectors is equal to Hk−1(j), and the result follows. + X n) / n − c > ε } = 0 n →∞ Chapter 8 137 Since X1 + ... + Xn is binomial with parameters n, x, we have ⎡ ⎛ X + ...
Therefore, the probability that at least one pair were both born on this date is approximately 1 − e−.6. ∑ 3. When n is large trials 1, ..., k are roughly independent. Conditioning on the amount of the initial check gives E[Return] = E[Return⎜A]/2 + E[Return⎜B]/2 = {AF(A) + B[1 − F(A)]}/2 + {BF(B) + A[1 − F(B)]}/2 = {A + B + [B − A][F(B) − F(A)]}/2
> (A + B)/2 where the inequality follows since [B − A] and [F(B) − F(A)] both have the same sign. (a) P{X ≥ 85} ≤ E[X]/85 = 15/17 (b) P{65 ≤ X ≤ 85} = 1 − P{⎜X − 75⎜ > 10} ≥ 1 − 25/100 ⎡⎤ (c) P ⎨ ⎢ 3. Chapter 3 28. Calculus yields that the maximum value of f(x)/g(x) = 60x3(1 − x)2 is attained when x = 3/5 and is thus equal to 1296/625. Then
P(E⎜V) = P (V E) P (E) P(V E) P(E) + P (V A) P(A) P(C G) P (G) 34. (a) P{both hit⎜at least one hit} = P{both hit} P{at least one hit} = p1p2/(1 − q1q2) Qi = 1 − pi, and we have assumed that the outcomes of the shots are independent. (a) P(W⎜A) = = P(A W) P(W) P(A W) P(W) + P(A M) P (M)
where M = Wc .48(.38) ≈ .443 .48(.38) + .37(.62) Therefore, 44.3 percent of the attendees were women. n − 2i, i = 0, 1, ..., n 6. 5! f X (3) (x)dx = x 2 (1 − x) 2 dx 2 !2 ! 1 / 4 1/ 4 49. No—they are conditionally independent given the coin selected. If s units are stocked and the demand is X, then the profit, P(s), is given by P(s) = bX − (s − X)P = sb if X
≤ s if X > s Hence E[P(s)] = ∫ s 0 (bx − (s − x)A) f (x)dx + = (b + A) ∫ s 0 xf (x)dx − sA = sb + (b + A) ∫ ∫ ∫ ∞ s sbf (x)dx f (x)dx + sb⎡⎤ 1 − 0⎣ s ∫ s 0 f (x)dx ⎤⎦ s 0 (x − s) f (x)dx Differentiation yields s s d d E[P(s)] = b + (b + A) ⎡⎤ xf (x) dx − s f (x)dx ⎤⎦ 0 ds ds⎣ 0 ⎦ s = b + (b + A) ⎡⎤ sf (s) − sf (s) − f (s)dx ⎤⎦ 0 ⎣ ∫ ∫ ∫ = b − (b + A) Chapter 5
∫ s 0 f (x)dx 65 Equating to zero shows that the maximal expected profit is obtained when s is chosen so that b b+A F(s) = where F(s) = 10. Generate random numbers U1, ..., Un, and approximate the integral by [k(U1) + ... + k(Un)]/n. P(E) . P{type j before type 1} = P{j⎜j or 1} = 20. ⎛ n + r − 1⎞⎢ ⎥by Proposition 6.2. ⎝ n − 1 ⎠Chapter 1 23. The
expected amount of money you will get is E[money] = 1 1 500 + 2000 = 1250 2 2 (b) Do not immediately buy but use your money to buy after one week. Solutions Manual A First Course in PROBABILITY Seventh Edition Sheldon Ross Prentice Hall, Upper Saddle River NJ 07458 Loading PreviewSorry, preview is currently unavailable. ⎝3⎠⎛15 ⎞(b)
there are⎢ ⎥⎝2⎠⎛14 ⎞there are⎢ ⎥⎝2⎠⎛13 ⎞there are⎢ ⎥⎝2⎠⎛13 ⎞there are⎢ ⎥⎝2⎠investments only in 1, 2, 3 investments only in 1, 2, 4 investments only in 1, 3, 4 investments only in 2, 3, 4 ⎛15 ⎞⎛14 ⎞⎛13 ⎞⎛12 ⎞⎢ ⎥+⎢ ⎥+ 2⎢ ⎥+⎢ ⎥= 552 possibilities ⎝2⎠⎝2⎠⎝2⎠⎝3⎠4 Chapter 1 Theoretical Exercises 2. Let Ei = {no type i in first n selections} ⎛N ⎞P{T > n}
= P⎢ ∪ Ei ⎥⎝ i=1 ⎠= ∑ (1 − Pi) n − ∑∑ (1 − P − P) + ∑∑∑ (1 − p i j n I n} 3. The weak law of large numbers yields that lim P{ ⎛ X 1 + ... Now estimate π by fD where f is the fraction of times the needle intersects one of the lines. Also, let Y denote the value that shows up on the die. P(A2⎜A1) = = = P(A1 A2) P(A1) P(A1 A2 M)α + P(A1 A2 F)(1 − α
) P(A1 M)α + P(A1 F)(1 − α) pm2 α + p 2f (1 − α) pmα + p f (1 − α) Hence, we need to show that pm2 α + p 2f [1 − α) > (pmα + pf(1 − α))2 or equivalently, that pm2 (α − α 2) + p2f (α − α 2) > 2α(1 − α)pfpm Chapter 3 27 Factoring out α(1 − α) gives the equivalent condition pm2 + p 2f > 2 pf m or (pm − pf)2 > 0 which follows
because pm ≠ pf. Then X is binomial with mean 65 and standard deviation 65(.35) ≈ 4.77. (a) 305 (b) 30 · 29 · 28 · 27 · 26 13. 6. Intuitively, the inequality follows because given the information that the policyholder had a claim in year 1 makes it more likely that it was a type policyholder having a larger claim probability. X= ∫ X (t)dt and taking
expectations gives 0 ∞ E[X] = ∫ ∞ ∫ E[X (t)] dt = P{ X > t}dt 0 7. (b) H2(1) = H1(1) = 1 H2(2) = H1(1) + H1(2) = 3 H2(3) = H1(1) + H1(2) + H1(3) = 6 H2(4) = H1(1) + H1(2) + H1(3) + H1(4) = 10 H2(5) = H1(1) + H1(2) + H1(3) + H1(4) + H1(5) = 15 H3(5) = H2(1) + H2(2) + H2(3) + H2(4) + H2(5) = 35 16. ⎛ 6⎞⎛ 2⎞⎛ 6⎞⎛ 6⎞⎛ 6⎞⎢ ⎥+⎢ ⎥ ⎢ ⎥ ⎥+⎢ ⎥
⎝5⎠⎝ 1⎠⎝ 4⎠⎝5⎠⎝3⎠7! = 35. Varr(Y) = 84.5 (a) E[(2 + X)2] = Var(2 + X) + (E[2 + X])2 = Var(X) + 9 = 14 (b) Var(4 + 3X) = 9 Var(X) = 45 39. P{ X = k + 1} P{ X = k} ⎛ Np ⎞⎛ N − np ⎞⎢ ⎥ ⎥k + 1⎠⎝ n − k − 1⎠⎝ = ⎛ Np ⎞⎛ N − Np ⎞⎢ ⎥ ⎥⎝ k ⎠⎝ n − k ⎠= Chapter 4 (Np − k)(n − k) (k + 1)(N − Np − n + k + 1) 61 30. Let A be the event that a randomly chosen
person is a cigarette smoker and let B be the event that she or he is a cigar smoker. (d) E[RS] = E[E[RS⎜S]] = E[SE[R⎜S]] = E[S 2] = μ2 + σ2 Cov (R, S) = μ2 + σ2 − μ2 = σ2 75. (1 − 1)n = n ⎛n⎞∑⎢ i ⎥(−1) n −1 i =0 14. Also, let A be the event that she is accepted and R that she is rejected. P {N ≥ n } P {X 1 ≥ X 2 ≥ ... ≥ X n } = ∞ E[N] = ∑ n =1
104 P{N ≥ n} = ∞ 1 n! 1 ∑ n! = e n =1 Chapter 7 Chapter 10 141 5. Choose a person at random P{they have accident} = P{acc. Let E be the event the child selected is the eldest, and let Fj be the event that the family has j children. Then the desired probability is 1− (n −1) d 1− (n − 2) d 1− 2 d n! ∫ ∫ ∫ ... n(n − 1) · · · (n − r + 1) = n!/(n − r)!
4. P{X = k} = k −1 n ∏ k −2 i =0 n−i ,k>1 n ⎛n⎞k⎢ ⎥n −1 ⎝ k ⎠= n2 E[X] = n 2n − 1 k =0 2 − 1 n 34. Hence, the probability is (1 + 2 + 3 + 4 + 5)/36 = 5/12. P{ace} = P{ace⎜interchanged selected} 1 27 +P{ace⎜interchanged not selected} =1 26 27 1 3 26 129 + = . Hence, Cov(X, Y) = 2 − 1 = 1. ⎛ 4⎞4⎢ ⎥(1 / 2) = 3/8 ⎝ 2⎠10 41. P{min(X1, ..., Xn) > t}
= P{X1 > t, ..., Xn > t} = e−λt...e−λt = e−nλt thus showing that the minimum is exponential with rate nλ. Let Xi,j , i ≠ j equal 1 if i and j form a matched pair, and let it be 0 otherwise. Hence, Var(X) = E[X2] −(E[X])2 ≤ E[cX − (E[X])2] = cE[X] − (E[X])2 = E[X](c − E[X]) = c2[α(1 − α)] where α = E[X]/c ≤ c2/4 where the last inequality first uses the
hypothesis that P{0 ≤ X ≤ c} = 1 to calculate that 0 ≤ α ≤ 1 and then uses calculus to show that maximum α(1 − α) = 1/4. 14 −5 / 2 e 4 10 , y > 10. P{rejected⎜1 defective} = 3/10 ⎛ 6 ⎞⎛10 ⎞P{rejected⎜4 defective} = 1 − ⎢ ⎥⎢ ⎥= 5/6 ⎝3⎠⎝ 3 ⎠5 3 6 10 P{4 defective⎜rejected} = = 75/138 5 3 3 7 + 6 10 10 10 81. (d) From (b) and (c) the (limiting)
number of molecules in urn 1 should have a binomial distribution with parameters (M, 1/2). ∞ ∞ ∑ i P{N > i} = ∑ i ∑ P{N = k} i =0 i = 0 k = i +1 ∞ = ∑ k =1 ∞ = k−1 ∑i P{N = k} i =0 ∑ P{N = k}(k − 1)k / 2 k =1 ∞ ⎛ ∞ ⎞= ⎢ ⎥ k 2 P{N = k} − kP{N = k} ⎥2 k =1 ⎠⎝ k =1 ∑ Chapter 4 ∑ 57 8. The result is true when n = 0, so assume that P{N(x) ≥ n} =
xn/(n − 1)! Now, 1 P{N(x) ≥ n + 1} = ∫ P{N (x) ≥ n + 1 U = y}dy 1 0 x = ∫ P{N (x − y) ≥ n}dy 0 x = ∫ P{N (u) ≥ n}du 0 x ∫ = u n −1 /(n − 1)! du by the induction hypothesis 0 = xn/n! which completes the proof. 43. + E[X ∑ X = nE [X ∑ X = x] x = E[X1 + ... + Xn⎜X1 + ... + Xn = x] = E X 1 i 1 n i =x] i Hence, E[X1⎜X1 + ... + Xn = x] = x/n 30.
P{acc.⎜no acc.} = 40. p. P(X = −3) = 1/8, P{X = 1} = 3/8, P{X = −1} = 3/8, P{X = −3} = 1/8 8. Hence, it will be heads with probability 1 − p. E[X − Y] = 1 m2 m m ∑∑ i − j . ⎛n⎞(i) ⎢ ⎥ ⎝ k ⎠2 ⎝ k ⎠(ii) n2n − 1 since there are n possible choices for the combined chairperson and secretary and then each of the other n − 1 can either be on or off the
committee. Let φ(s, t) = E[esX+tY] ∂2 φ (s, t) s = 0 = E[XYesX + tY] s = 0 = E[XY] ∂s∂t t =0 t =0 ∂ φ (s, t) s = 0 = E[X], ∂s t =0 ∂ φ (s, t) s = 0 = E[Y] ∂t t = 0 53. The result follows from the identity E[XY] = E[E[XY⎜X]] = E[XE[Y⎜X]] which is obtained by noting that, given X, X may be treated as a constant. Then lim P (X < bn } = P{X ≤ b} ≠
h→0 bn → b P{X < b}. n ∏[1 − P(E)] i 1 (a) They will all be white if the last ball withdrawn from the urn (when all balls are withdrawn) is white. P{at least one 6 ⎜ sum of 12} = 1. If Wn is the total weight of n cars and A is the amount of weight that the bridge can withstand then Wn − A is normal with mean 3n − 400 and variance .09n + 1600. 7. ⎛ 7 ⎞
⎝10 ⎞(a)⎢ ⎥ ⎥= 1/12 ≈ .0833 ⎝5⎠⎝ 5 ⎠⎛ 7 ⎞⎛ 3 ⎞⎛10 ⎞(b)⎢ ⎥ ⎥ ⎥+ 1/12 = 1/2 ⎝ 4 ⎠⎝ 1 ⎠⎝ 5 ⎠38. P(Bj⎜Ni) = P(N i B j) P(B j) P(N i Bi) P(Bi) + P(N i Bic) P(Bic) Pj if j ≠ i (1 − α i) Pi + Pi (1 − α i) Pj if j = i (1 − α i) Pi + 1 − Pi 5. Also, let B be such that ⎜g(x) ⎜ < B. From the information of the problem we can conclude that both of Smith's
parents have one blue and one brown eyed gene. 1 4 ∫∫ dydx = (area of circle)/4 = π/4. (a) 1 < 2 < 3, 1 < 3 < 2, 2 < 1 < 3, 2 < 3 < 1, 3 < 1 < 2, 3 < 2 < 1, 1 < 2 = 3, 1 < 3 = 2, 2 < 1 = 3, 2 < 3 = 1, 3 < 1 = 2, 3 < 2 = 1, 1 = 2 < 3, 1 = 3 < 2, 2 = 3 < 1, 1 = 2 = 3 ⎛ n⎞(b) The number of outcomes in which i players tie for last place is equal to⎢ ⎥, the number ⎝i⎠of ways to choose these
i players, multiplied by the number of outcomes of the remaining n − i players, which is clearly equal to N(n − i). Assuming that X is approximately distributed as a binomial random variable with parameters n = 200, p = .12, then, with Z being a standard normal random variable, ⎡⎤ X − 200(.12) 19.5 − 200(.12) ⎤⎥ P{X > 19.5} = P ⎨ > ⎬ 200(.12)(.88)
⎣⎦ 200(.12)(.88) ≈ P{Z > −.9792} ≈ .8363 29. J = 1 1 x1 0 e = − e x1 = −y2 x1 = log y2, x2 = y1 − log y2 1 − λ log y 2 − λ (y1 − log y 2) λe λe y2 1 2 − λy1 = λ e , 1 ≤ y2, y1 ≥ log y2 y2 fY1 ,Y2 (y1 , y2) = 58. Chapter 7 125 18. It is easy to see that the nth derivative of ∞ ∑ (t 2 / 2) j / j ! will, when evaluated at t = 0, equal 0 j =0 whenever n is
odd (because all of its terms will be constants multiplied by some power of t). should be 83. ⎡1 pair i consists of a man and a woman (a) Let Xi = ⎨ ⎩0 otherwise 10 19 E[XiXj] = P{Xi = 1, Xj = 1} = P{Xi = 1}P{Xj = 1⎜X2 = 1} 10 9 = ,i≠j 19 17 E[Xi] = P{Xi = 1} = ⎡ 10 ⎤100 E⎢ Xi ⎥ = ⎣ 1 ⎦19 ∑ ⎛ Var⎢⎝ 10 ∑ 1 ⎡10 9 ⎛ 10 ⎞2 900 18 ⎤10 ⎛ 10 ⎞−⎢ ⎥= X i
⎥= 10 ⎢1 − ⎥+ 10 · 9⎢ 2 19⎢ 19 ⎠⎣19 17 ⎝ 19 ⎠⎦ (19) 17 ⎡1 pair i consists of a married couple (b) Xi = ⎨ ⎩0 otherwise E[Xi] = 1 1 1 , E[XiXj] = P{Xi = 1}P{Xj = 1⎜Xi = 1} = , i≠j 19 19 17⎡ 10 ⎤10 E⎢ Xi ⎥ = ⎣ 1 ⎦19 ∑ ⎛ Var⎢ ⎢⎝ 43. ⎡1 if couple j are seated next to each other Let Xj = ⎨ ⎩0 otherwise⎡ 10 ⎤2 2 20 (a) E⎢ X j ⎥ = 10 = ; P{Xj = 1} = since
there are 2 people seated next to wife j 19 19 19⎣ 1 ⎦2 and so the probability that one of them is her husband is . 63. It is the largest possible since 1.8 = P{ X i = 1} = 3P{ X i = 1} . If X has distribution function F and density f, then for a > 0 f(t) 1/ a 1 = = ,0 v which is clearly increasing when β ≥ 1 and decreasing otherwise. 2 · 9! − 228! since 2 ·
9! is the number in which the French and English are next to each other and 228! the number in which the French and English are next to each other and the U.S. and Russian are next to each other. m i =1 ni ⎛ n⎞There are⎢ ⎥different 0 − 1 vectors whose sum is j, since any such vector can be ⎝ j⎠characterized by a selection of j of the n indices
whose values are then set equal to 1. exp(−10e−5) 69. 10 15 1 10 5 1 4 + 30 30 2 30 30 2 Chapter 3 P(C G) P(G) + P(C G c) P(G c) = (1 / 2)(2 / 5) = 5/11 (1 / 2)(2 / 5) + (2 / 5)(3 / 5) = 54/62 25 36. Chapter 6 89 5. Let N be the number of games played. (a) P{silver in other⎜silver found} = P{S in other, S found} . Let Ni denote the event that the ball
is not found in a search of box i, and let Bj denote the event that it is in box j. ⎝k ⎠13. Suppose X ≥st Y and f is increasing. Consider pi + p j a new p-vector with all other pk, k ≠ i, j, as before and with pi = p j = . Let L, L < D, denote the length of the needle. (c) E[X] = 1[18/38 + (20/38)(18/38)2] − [(20/38)2(20/38)(18/38)] − 3(20/38)3 ≈ −.108 21. a b
+ = 3 / 5 . P1 g and 1 b ⎜ at least one b} = 20 1/ 2 = 2/3 3/ 4 Chapter 3 8. Hence, its expected value is m/(1 − p). Let s be the initial price of the stock. Then for any a E[(X − a)2 = E[(X − μ + μ − a)2] = E[(X − μ)2] + (μ − a)2 + 2E[(x − μ)(μ − a)] = E[(X − μ)2] + (μ − a)2 + 2(μ − a)E[(X − μ)] = E[(X − μ)2 + (μ − a)2 2. Let N denote the number of
games played. Cov(Yn, Yn) = Var(Yn) = 3σ2 Cov(Yn, Yn+1) = Cov(Xn + Xn+1 + Xn+2, Xn+1 + Xn+2 + Xn+3) = Cov(Xn+1 + Xn+2, Xn+1 + Xn+2) = Var(Xn+1 + Xn+2) = 2σ2 Cov(Yn, Yn+2) = Cov(Xn+2, Xn+2) = σ2 Cov(Yn, Yn+j) = 0 when j ≥ 3 40. (a) P{x > 0} = P{win first bet} + P{lose, win, win} = 18/38 + (20/38)(18/38)2 ≈ .5918 (b) No,
because if the gambler wins then he or she wins $1. Hence, Boole's inequality we have that ⎛n⎞P(∪ Bi) ≤ ⎢ ⎥[1 − (1 / 2) k]n − k⎝k ⎠⎝n⎠⎝n⎠Hence, if⎢ ⎥[1 − (1 / 2) k]n − k < 1 then there is a positive probability that none of the ⎢ ⎥k ⎠k ⎠events Bi occur, which means that there is a positive probability that for every set of k contestants there is a
contestant who beats each member of this set. p1 = 4/20, p2 = 8/20, p3 = 5/20, p4 = 2/20, p5 = 1/20 (a) (b) There are a total of 4 · 1 + 8 · 2 + 5 · 3 + 2 · 4 + 1 · 5 = 48 children. ∑ ∑ Let S denote the number of heads that occur when all n coins are tossed, and note that S has a distribution that is approximately that of a Poisson random variable with
mean λ. Because only one of the cards matches the age of the person i⎡1000 ⎤1000 E⎢ Xi ⎥ = E[X i] = 1⎣ i =1 ⎦ i =1 ∑ 14. (a) (b) (c) (d) S = {(1, g), (0, g), (1, f), (0, f), (1, s), (0, s)} A = {(1, s), (0, s)} B = {(0, g), (0, f), (0, s)} {(1, s), (0, s), (1, g), (1, f)} 7. 52 (a) e−2.2 (b) 1 − e−2.2 − 2.2e−2.2 = 1 − 3.2e−2.2 Chapter 4 55. E[XY] = ∫∫ y 2e 0 0 ∞ = ∫ 0
108 −2x dydx x 2e − 2 x dx = ∞ Γ(3) 1 1 2 −y y e dy = = 80 8 4 ∫ Chapter 7 ∞ E[X] = ∫ x xf x (x)dx, f x (x) = 0 ∫ ∞ yfY (y)dy, fY (y) = 0 ∞∞ = ∫∫ y 2e −2 x dxdy x y 2e −2 x dydx x 0 ∞x = ∫∫ 0 0 ∞ = ∫ 1 2 ∞ E[Y] = 2e −2 x dy = 2e−2x dx x 0 ∫ xe −2x dx = 0 Cov(X, Y) = 2e −2 x dx x 0 ∫ 1 Γ (2) 1 ye− 2 dy = = 4 4 4 ∫ 1 1 1 1 − = 4 24 8 39. ∑⎡ 50 ⎤
Letting Xi denote the ith roundoff error it follows that E⎢ X i ⎥ = 0, ⎣ i =1 ⎦50 ⎛ ⎞Var⎢ X i ⎥= 50 Var(X1) = 50/12, where the last equality uses that .5 + X is uniform (0, 1) ⎝ i =1 ⎠and so Var(X) = Var(.5 + X) = 1/12. (a) 1 − 7 ∑e −4 i 4 / i! ≡ p i =0 (b) 1 − (1 − p)12 − 12p(1 − p)11 (c) (1 − p)i−1p 65. 131 51. (a) P(R ∪ N) = 1 − .6 = .4 (b) .4 = P(R ∪ N)
= P(R) + P(N) − P(RN) = .2 + .3 − P(RN) Thus, P(RN) = .1 11. Now, .10 = P{Z > (c − 12)/2}. ⎝i⎠n n −1 n ⎛n⎞⎝n⎠Tn+1 = ⎢ ⎥Tn − i = 1 + ⎢ ⎥Tn − i = 1 + ⎢ ⎥Tk . Then ⎡⎛ X − v ⎞β ⎤ ⎡ X−v⎤ ≤ x1 / β ⎬ P ⎨⎢ ≤ x⎬ = P⎨ ⎢ ⎝ α ⎠⎦ ⎣ ⎝ α ⎠⎦ = P{X ≤ v + αx1/β} = 1 − exp{−x}. The joint density of the point (X, Y) at which the accident occurs is 1 , −3/2 < x, y <
3/2 9 = f(x) f(y) f(x, y) = where f(a) = 1/3, −3/2 < a < 3/2. n n! F k −1 (x) f (x)[1 − F (x)]n − k by k = i + 1 − − (n k) ! (k 1) ! k = j + 1 ∑ n! F j −1 (x) f (x)[1 − F (x)]n − j (n − j)!(j − 1)! f X (n +1) (x) = (2n + 1)! n x (1 − x) n n !n ! In order for X(i) = xi , X(j) = xj , i < j , we must have (i) i − 1 of the X's less than xi (ii) 1 of the X's equal to xi (iii) j − i
− 1 of the X's between xi and xj (iv) 1 of the X's equal to xj (v) n − j of the X's greater than xj Hence, f x(i) , X (j) (xi , x j) = Chapter 6 n! F i −1 (xi) f (xi)[F (x j) − F (xi)] j −i −1 f (x j)[1 − F(xj)]n−j (i − 1)!1!(j − i − 1)!1!(n − j)! 95 29. Since Xi + Ri has mean 100 × 5 + 99 × .25 = i =1 ⎜ i =1 ⎜ 524.75 and variance 2500 + 99/48 = 2502 it
follows that the desired probability is approximately equal to P{N(0, 1) ≤ [550 − 524.75]/(2502)1/2} = P{N(0, 1) ≤ .505} = .693 It should be noted that the above used that ∑ ∑ ∑ ∑ ⎛1 ⎞Var(Ri) = Var⎢ Unif [0,1] ⎥= 1/48 2 ⎝ ⎠9. i =4 Chapter 4 51 48. Now each size ordering of the n + m − 1 elements under consideration is equally likely and so the
probability that the m smallest are the ones in row i is 1 ⎛ n + m − 1⎞⎢ ⎥. Hence, E[Y] = 1, E[X] = E[E[X⎜Y]] = E[Y] = 1 Since, E[XY] = E[E[XY⎜Y]] = E[YE[X⎜Y]] = E[Y2] = 2 (since Y is exponential with mean 1, it follows that E[Y2] = 2). k k =1 Chapter 7 ∞ 6. ∫ (a) c (1 − x 2)dx = 1 ⇒ c = 3 / 4 −1 x 3 3⎛ x3 2 ⎞(1 − x 2) dx = ⎢⎢ x − + ⎥⎥, −1 < x < 1 (b)
F(x) = 4 −1 4⎝ 3 3⎠∫ 2. Hence, to obtain all 13 hearts we would expect to turn over from Example 3j, 14 13 53 52 − +1= (53). ai ∏ (1 − a j) is the probability that the first head appears on the ith flip and ∞ ∏ (1 − a) is the i =1 j =1 probability that all flips land on tails. Now in the interval 0 < Y < 5. The result follows since (a) there are n2n − 1
selections in which the chair, secretary and treasurer are the same person. Now, e−x = 1/2 when x = log(2). ⎝ 3⎠⎝ 2⎠20. 12 Chapter 2 21. If check 1, then (if desired) 2: Expected Cost = C1 + (1 − p)C2 + pR1 + (1 − p)R2; if check 2, then 1: Expected Cost = C2 + pC1 + pR1 + (1 − p)R2 so 1, 2, best if p C2 C1 + (1 − p)C2 ≤ C2 + pC1, or C1 ≤ 1− p
30. Let Xi equal 1 if a changeover occurs on the ith flip and 0 otherwise. P{X = i} = 1 ⎛n⎞P{ X = i p}dp = ⎢ ⎥p i (1 − p) n − i dp i 0 0⎝ ⎠∫ ∫ ⎛ ⎛ n ⎞i !(n − i)! = 1 /(n + 1) = ⎢ ⎥⎝ i ⎠(n + 1)! 118 Chapter 7 72. (a) 1 ⎛10 ⎞7 3 1 ⎛ 10 ⎞7 3 ⎢ ⎥.4 .6 + ⎢ ⎥.7 .3 2⎝ 7 ⎠2⎝ 7 ⎠1 ⎛9⎞7 3 1 7 3 ⎢ ⎥.4 .6 + .7 .3 2 ⎝ 6 ⎠2 (b) .55 50. Hence, (where T0 = 1). The event (∪∞n =1
En)c is the event that 6 never appears. The number of integer solutions of x1 + ... + xr = n, xi ≥ mi is the same as the number of nonnegative solutions of y1 + ... + yr = n − r ∑ m , y ≥ 0. Binomial with parameters m and Pi + Pj. (b) Using (a) we have that Var(Ni + Nj) = m(Pi + Pj)(1 − Pi − Pj) and thus m(Pi + Pj)(1 − Pi − Pj) = mPi(1 − Pi) + mPj(1 −
Pj) + 2 Cov(Ni, Nj) Simplifying the above shows that Cov(Ni, Nj) = −mPiPj. 19. (i) 1/365 (ii) 1/365 (iii) 1 The events, though independent in pairs, are not independent. 88 n ∑x i =k i =1 Chapter 6 Theoretical Exercises 1. ∑⎢⎝ i ⎥(1/ 2) 10 i =7 42. Therefore, three out of every four sons will be eldest sons. Hence (a) P{X + Y = 2} = P{X = 0)P{Y = 2} +
P{X = 1}P{Y = 1} + P{X = 2}P{Y = 0} ⎛10 ⎞⎛10 ⎞= e − 2⎢ ⎥(3 / 4) 2 (1 / 4)8 + 2e − 2⎢ ⎥(3 / 4)(1 / 4)9 + 2e − 2 (1 / 4)10 ⎝1⎠⎝2⎠(b) P{XY = 0} = P{X = 0} + P{Y = 0} − P{X = Y = 0} = e−2 + (1/4)10 − e−2(1/4)10 (c) E[XY] = E[X]E[Y] = 2 · 10 · Chapter 7 3 = 15 4 119 77. 18. 53. (a) e−1 (b) e−1/2 33. Let M and F denote, respectively, the events that
the policyholder is male and that the policyholder is female. 11 1 5 + 42 2 (c) 1 38. 8 ∑ E (Y) i 1 2 3 3 147 +8 = 11 20 120 110 Number the small pills, and let Xi equal 1 if small pill i is still in the bottle after the last large pill has been chosen and let it be 0 otherwise, i = 1, ..., n. (a) It is binomial with parameters n − 1 and p. Each kitten can be
identified by a code number i, j, k, l where each of i, j, k, l is any of the numbers from 1 to 7. We will show by induction on n that ⎡⎛ a − t ⎞n ⎥ ⎥P{X(k) − X(k−1) > t} = ⎨⎝ a ⎠⎜ ⎩0 if t < a if t > a It is immediate when n = 1 so assume for n − 1. (a) P{husband under 25} = (212 + 36)/500 = .496 (b) P{wife over⎜husband over} = P{both over}/P{husband
over} = (54 / 500) (252 / 500) = 3/14 ≈ .214 (c) P{wife over⎜husband under} = 36/248 ≈ .145 22. (a) P{X + Y ≥ 110} = P{X + Y ≥ 109.5} ⎡ X + Y − 97.6 109.5 − 97.6 ⎤ ≥ = P⎨ ⎬ 73.76 73.76 ⎦ ⎣ ≈ P{Z > 1.3856} ≈ .0829 (b) P{Y ≥ X} = P{Y − X ≥ −.5} ⎡ Y − X − (−3.2) − .5 − (−3.2) ⎤ ≥ = P⎨ ⎬ 73.76 73.76 ⎦ ⎣ ≈ P{Z ≥ .3144} ≈ .3766 35. Hence, from
part (a) and the preceding we obtain that for i ≠ j, ρ(Di, Dj) = Chapter 7 p (1 − p) 1 = (n − 1) p (1 − p) n − 1 111 48. Let Xi equal 1 if trial i is a success and 0 otherwise. Now, i i =1 E[Xi] = P{small pill i is chosen after all m large pills} = 1/(m + 1) E[Yi] = P{ith created small pill is chosen after m − i existing large pills} = 1/(m − i + 1) Thus, m (a)
E[X] = n/(m + 1) + ∑1/(m − i + 1) i =1 (b) Y = n + 2m − X and thus E[Y] = n + 2m − E[X] 25. 3. (b) P(AcB) = P(B) − P(AB) = .07 − .05 = .02. 2 0 ∫ 2 46. 72 Chapter 5 Var (aX + b) = a 2σ 2 = a σ 7. = 24 4 By symmetry P{D win} = P{G win} and as the probabilities must sum to 1 we have. The probability that a package will be returned is p = 1 −
(.99)10 − 10(.99)9(.01). Otherwise twice the probability given in Problem 2. Also, let Z denote a standard normal random variable. Each roll that is either a 7 or an even number will be a 7 with probability p= P (7) 1/ 6 = = 1/4 P(7) + P(even) 1 / 6 + 1 / 2 Hence, from Example 4i we see that the desired probability is 7 ⎛7⎞∑⎢ i ⎥(1/ 4) (3 / 4) i 7 −i = 1
− (3/4)7 − 7(3/4)6(1/4) i =2 80. EFG = FG. If T is the number of tests needed for a group of 10 people, then E[T] = (.9)10 + 11[1 − (.9)10] = 11 − 10(.9)10 35. P(B) = P(B⎜C)P(C) + P(B⎜Cc)P(Cc) 19 1 29 29 = 0 + = 48 32 48 1536 29. P(S ∪ H ∪ D ∪ C) = P(S) + P(H) + P(D) + P(C) − P(SH) − ... − P(SHDC) ⎛ 39 ⎞⎛ 26 ⎞⎛13 ⎞4⎢ ⎥6⎢ ⎥4⎢ ⎥13 13 13 = ⎝ ⎠− ⎝
⎠+ ⎝ ⎠⎛ 52 ⎞⎛ 52 ⎞⎛ 52 ⎞⎢ ⎥⎢ ⎥⎢ ⎥⎝ 13 ⎠⎝ 13 ⎠⎝ 13 ⎠⎛ 39 ⎞⎛ 26 ⎞4⎢ ⎥− 6⎢ ⎥+ 4 13 13 = ⎝ ⎠⎝ ⎠⎛ 52 ⎞⎢ ⎥⎝ 13 ⎠55. E = EF ∪ EFc 18 Chapter 2 15. Hence, n2n − 1 also represents the desired quantity. 33 33 9 21 = 3 3 = 1/2. By induction: true when t = 0, so assume for t − 1. Summing over i ⎝ k − 1 ⎠yields the number of subsets of size k. (a) Let Xi equal 1 if
the person in position i is a man who has a woman next to him, and let it equal 0 otherwise. Let qx = 1 − px (a) P{A not hit} = P{A not hit⎜at least one is hit} = P{A not hit, B hit}/P{at least one is hit} = qBpA/(1 − qAqB) (b) P{both hit} = P{both hit⎜at least one is hit} = P{both hit}/P{at least one hit} = pApB/(1 − qAqB) (c) (qAqB)n − 1(1 − qAqB)
(d) P{n rounds⎜A unhit} = P{n rounds, A unhit}/P{A unhit} (q AqB) n −1 p AqB = qB p A /(1 − q AqB) = (qAqB)n − 1(1 − qAqB) 32 Chapter 3 (e) P(n rounds⎜both hit} = P{n rounds both hit}/P{both hit} (q AqB) n −1 p A pB = pB p A /(1 − q AqB) = (qAqB)n−1(1 − qAqB) Note that (c), (d), and (e) all have the same answer. Hence, from the above
F(a2, b2) = P{a1 < X ≤ a2, b1 < Y ≤ b2} + F(a1, b2) − F(a1, b1) + F(a2, b1) − F(a1, b1) + F(a1, b1). ⎝ 3 ⎠⎝ 3⎠⎝ 1 ⎠⎝ 2⎠⎝3 ⎠2 Chapter 1 ⎛ 7 ⎞⎛ 5⎞⎛ 7 ⎞⎛ 5⎞⎛ 7 ⎞⎛ 5 ⎞⎛ 7 ⎞⎛ 5⎞(c) There are⎢ ⎥⎢ ⎥+⎢ ⎥⎢ ⎥+⎢ ⎥⎢ ⎥= 910 possible committees. 5. With Pj equal to the probability that 4 consecutive heads occur within j flips of a fair coin, P1 = P2 = P + 3 = 0, and P4 =
1/16 P5 = (1/2)P4 + 1/16 = 3/32 P6 = (1/2)P5 + (1/4)P4 + 1/16 = 1/8 P7 = (1/2)P6 + (1/4)P5 + (1/8)P4 + 1/16 = 5/32 P8 = (1/2)P7 + (1/4)P6 + (1/8)P5 + (1/16)P4 + 1/16 = 6/32 P9 = (1/2)P8 + (1/4)P7 + (1/8)P6 + (1/16)P5 + 1/16 = 111/512 P10 = (1/2)P9 + (1/4)P8 + (1/8)P7 + (1/16)P6 + 1/16 = 251/1024 = .2451 The Poisson approximation gives
P10 ≈ 1 − exp{−6/32 − 1/16} = 1 − e−.25 = .2212 54 Chapter 4 70. Because X is binomial with parameters 1000, .52, we have ⎡⎤ X − 1000(.52) 469.5 − 1000(.52) ⎤⎥ P{X > 469.5} = P ⎨ > ⎬ 1000(.52)(.48) ⎦⎦ ⎣ 1000(.52)(.48) ≈ P{Z > −3.196} ≈ .9993 30. e−λt + (1 − e−λt)p 71. ⎛ 6⎞⎛ 7⎞⎛ 4⎞(a)⎢ ⎥+⎢ ⎥+⎢ ⎥= 42 possibilities. (a) E[X2 + 4X + 4] =
E[X2] + 4E[X] + 4 = Var(X) + E2[X] + 4E[X] + 4 = 14 (b) Var(4 + 3X) = Var(3X) = 9Var(X) = 45 34. 19 ∑ (b) For i ≠ j, E[XiXj] = P{Xi = 1, Xj = 1} = P{Xi = 1}P{Xj = 1⎜Xi = 1} 2 2 since given Xi = 1 we can regard couple i as a single entity. FY(x) = P{aX + b ≤ x} x − b⎤ ⎡ = P⎨ X ≤ ⎬ when a > 0 a ⎦ ⎣ = FX((x − b)/a) when a > 0. 49. P(C1C2⎜F) = =
P(C1C2 F) P(F) P(F C1C2) P(C1C2) p5 (p1 p2 + p3 p4 − p1 p2 p3 p4) p5 p1 p2 = p5 (p1 p2 + p3 p4 − p1 p2 p3 p4) 69. 9. 140 Chapter 9 Chapter 10 1. If X is the amount that you win, then P{X = 1.10} = 4/9 = 1 − P{X = −1} E[X] = (1.1)4/9 − 5/9 = −.6/9 ≈ =−.067 Var(X) = (1.1)2(4/9) + 5/9 − (.6/9)2 ≈ 1.089 36. (a) p(6) = 1 − (5/6)2 = 11/36,
p(5) = 2 1/6 4/6 + (1/6)2 = 9/36 p(4) = 2 1/6 3/6 + (1/6)2 = 7/36, p(3) = 2 1/6 2/6 + (1/6)2 = 5/36 p(2) = 2 1/6 1/6 + (1/6)2 = 3/36, p(1) = 1/36 (d) p(5) = 1/36, p(4) = 2/36, p(3) = 3/36, p(2) = 4/36, p(1) = 5/36 p(0) = 6/36, p(−j) = p(j), j > 0 46 Chapter 4 11. There were 8 · 2 · 9 = 144 possible codes. (The above used that f X (n) (s) = n⎢ ⎥⎝a⎠30. . ⎛5⎞
3 ⎛5⎞4 2 5⎢ ⎥(.2) (.8) + ⎢ ⎥(.2) (.8) + (.2) 3 4 ⎝ ⎠⎝ ⎠44. Let ⎡1 if a run of size k begins at the j th flip Ij = ⎨ ⎩0 otherwise Then n − k +1 Number of runs of size k = ∑I j j =1 Chapter 7 123⎡ n − k +1 ⎤E[Number of runs of size k = E⎢ Ij⎥⎣ j =1 ⎦⎦∑ n−k = P(I1 = 1) + ∑ P(I j = 1) + P (I n − k +1 = 1) j =2 = pk(1 − p) + (n − k − 1)pk(1 − p)2 + pk(1 − p) 10.
26. Hence, n−2 ∑X i the i =0 amount of time the light can operate is gamma distributed with parameters (n − 1, 2λ). If we let Xk equal 1 if Ak occurs and 0 otherwise then n X= ∑X k k =1 Hence, n E[X] = ∑ E[X k =1 k] = n ∑ P(A) k k =1 But n E[X] = ∑ k =1 122 P{ X ≥ k} = n ∑ P(C) . ⎛ 2n ⎞(e) Since the 2 players in game i are equally likely to be
any of the⎢ ⎥pairs it follows that ⎝2 ⎠⎝ 2n ⎞P(Bi) = 1 ⎢ ⎥. fcX(a) = (b) A chi-squared random variable with 2n degrees of freedom can be regarded as being the sum of n independent chi-square random variables each with 2 degrees of freedom (which by Example is equivalent to an exponential random variable with parameter λ). 5 · 4 · 3 12 = 5 · 5 ·
5 25 40. (a) P{Y1 = 1⎜Y2 = 1} = 2/12 = 1 − P{Y1 = 0⎜Y2 = 1} (b) P{Y1 = 1⎜Y2 = 0} = 3/12 = 1 − P{Y1 = 0⎜Y2 = 0} 38. (a) P{min Xi ≤ a} = 1 − P{min Xi > a} = 1 − ∫ ∫ (b) P{max Xi ≤ a} = ∏ P{ X i ∏ P{ X i > a} = 1 − e −5λa ≤ a} = (1 − e − λa)5 2 50. 1 λ log 2 (a) all values in (a, b) (b) μ (c) 0 Chapter 5 73 14. (a) fX(x) = e−x , fY(y) = e−y, 0 < x
< ∞, 0 < y < ∞ P{X < Y} = 1/2 (b) P{X < a} = 1 − e−a 11. (b) P{X = i} = i/148, i = 40, 33, 25, 50 E[X] = [(40)2 + (33)2 + (25)2 + (50)2]/148 ≈ 39.28 E[Y] = (40 + 33 + 25 + 50)/4 = 37 22. ⎛ i − 1⎞4 i−4 P{wins in i games} = ⎢ ⎥(.6) (.4) 3 ⎝ ⎠73. Let R and N denote the events, respectively, that the student wears a ring and wears a necklace. p n dp = n
+ m +1 0 0 ∫ Chapter 3 ∫ 45 Chapter 4 Problems 1. P{0 ≤ X ≤ 40} = 1 − P{⎜X − 20⎜ > 20} ≥ 1 − 20/400 = 19/20 2. Let Xi be the indicator for the event that person i is given a card whose number matches his age. Let ⎡1 ball j removed before ball 1 Xj = ⎨ ⎩0 - - ⎡ ⎤E⎢ X j ⎥ = ⎢⎣ j ≠1 ⎦⎦∑ ∑ E[X j] = j ≠1 ∑ P{ball j before ball 1} j ≠1 = ∑ P{ j j or 1} j ≠1
= ∑W (j) / W (1) + W (j) j ≠1 3 21. 2 87 55. As P(Ai = 2/(2n − 1) it follows that, for n large, the number of wives that sit next to their husbands is approximately Poisson with mean 2n/(2n − 1) ≈ 1. 90. y FY(y) = P{eX ≤ y} = FX(log y) fY(y) = f X (log y) Chapter 5 1 1 = ,1 t}dt ∫ P{ X n > x n }nx n −1dx by t = xn, dt = nxn−1dx 0 ∞ = 0 ∞ = ∫ P{X >
x}nx n −1 dx 0 6. g g b P (RBG G last) = . (a) P(R) = P(R⎜strong)P(strong) + P(R⎜moderate)P(moderate) + P(R⎜weak)P(weak) = (.8)(.7) + (.4)(.2) + (.1)(.1) = .65 (b) P(strong⎜R) = P(R strong) P(strong) P(R) (.8)(.7) 56 = = .65 65 Similarly, P(moderate⎜R) = (c) c P(strong⎜R) = 8 1 , P(weak⎜R) = 65 65 P(R c strong) P(strong) P(Rc) (.2)(.7) 14 = =
.35 35 Similarly, P(moderate⎜Rc) = 52. P{X ≤ a2, Y ≤ b2} = P{a1 < X ≤ a2, b1 < Y ≤ b2} + P{X ≤ a1, b1 < Y ≤ b2} + P{a1 < X ≤ a2, Y ≤ b1} + P{X ≤ a1, Y ≤ b1}. Cov(X, Y) = Cov(W1 + W2 , W1 − W2) = Cov(W1, W1) − Cov(W2, W2) = Var(W1) − Var(W2) = 0 ∞x 38. Hence ⎝ m ⎠P{Xij is a saddlepoint} = 1 1 (m − 1)!(n − 1)! = (n + m − 1)! ⎛ n + m
− 1⎞m⎢ m ⎥⎝ ⎠and so P{there is a saddlepoint} = P⎛ ∪{ X ij is a saddlepoint}⎞⎥⎠i, j = P{ X ij is a saddlepoint} ∑ i, j = 22. 1 P{arrangement} p k (1 − p) n − k = = n⎠⎛ P{k successes} ⎛ n ⎞k n−k⎢ ⎥p (1 − p)⎢ ⎥⎝k ⎠⎝k ⎠Condition on the number of functioning components and then use the results of Example 4c of Chapter 1: n Prob = ⎛n⎞∑⎢ i ⎥⎠p (1
− p) i =0 i n − i⎡⎛ i + 1 ⎞⎛ n ⎞⎤⎥ n − i ⎥ i ⎥⎠⎝ ⎠⎦⎣⎛ i +1⎞where⎢ ⎥= 0 if n − i > i + 1. 70. 1 − 3 1 − 4.2 e + e 2 2 56. Each arrangement is determined by the choice of the r positions where the black balls are situated. Hence, q1 = 4/48, q2 = 16/48, q3 = 15/48, q4 = 8/48, q5 = 5/48 22. The situations in which a given steering committees vote is
decisive are as follows: given member for for against against other members of S.C. both for one for, one against both for one for, one against both for other council members 3 or 4 against at least 2 for at least 2 for 3 of 4 against P{decisive} = p34p(1 − p)3 + p2p(1 − p)(6p2(1 − p)2 + 4p3(1 − p) + p4) + (1 − p)2p(1 − p)(6p2(1 − p)2 + 4p3(1 − p) + p4) + (1
− p)p24p(1 − p)3. Assuming teachers are distinct. The joint mass function is P{Xi = xi, i = 1, ..., n} = 1/⎛ n ⎞⎥, xi ∈ {0, 1}, i = 1, ..., n, ⎝k ⎠As this is symmetric in x1, ..., xn the result follows. Let Ci be the event that coin i is being flipped (where coin 1 is the one having head probability .4), and let T be the event that 2 of the first 3 flips land on heads.
= 10 = 7 3 102 (.3) 2 + (.8) 2 10 10 Ei are conditionally independent given the guilt or innocence of the defendant. ⎤ ⎡ 100 P ⎨ X i > 525⎬ ≈ ⎦ ⎣ i =1 ∑ ⎡⎢ 525 − 500 ⎤⎥ P ⎨ N (0,1) > ⎬ = P{N (0,1) > .5} = .3085 (100 × 25) ⎥⎦ ⎢⎣ where the above uses that an exponential with mean 5 has variance 25. For 3!4! instance the arrangement r, r, u, u, r, r, u
specifies the path whose first 2 steps are to the right, next 2 steps are up, next 2 are to the right, and final step is up. ⎝ 2⎠⎝ 2⎠39. ⎝ j⎠∑ 6. a a (a) P{X > X(n)} = P{X is largest of n + 1} = 1/(n + 1) (b) P{X > X(1)} = P{X is not smallest of n + 1} = 1 − 1/(n + 1) = n/(n + 1) (c) This is the probability that X is either the (i + 1)st or (i + 2)nd or ... jth
smallest of the n + 1 random variables, which is clearly equal to (j − 1)/(n + 1). File loading please wait... The desired probability is 67. Hence, the number of such couples is approximately Poisson with mean 80,000/(365)2 ≈ .6. Hence the answer is 1/2. Let X1, ..., Xn be n independent uniform random variables over (0, a). P{X > 10} = 1 14. So
consider any ordering, and randomly choose a component in such a manner that each of the 47 components is equally likely to be chosen. X is uniform on (0, L). P{E has 3 ⎜ N − S has 8} = P{E has 3, N − S has 8} P{N − S has 8} ⎛13 ⎞⎛ 39 ⎞⎛ 5 ⎞⎛ 21⎞⎛ 52 ⎞⎛ 26 ⎞⎢ ⎥⎢ ⎥⎢ ⎥ ⎥ ⎥8 18 3 10 26 13 = ⎝ ⎠⎝ ⎠⎝ ⎠⎝ ⎠⎝ ⎠⎝ ⎠= .339 ⎛13 ⎞⎛ 39 ⎞⎛ 52 ⎞⎢ ⎥⎢ ⎥⎢ ⎥⎝ 8 ⎠⎝18 ⎠⎝ 26
⎠4. (Its value when n = 5 is .3819) 47. j j n 36 Let Wi, i = 1, 2, denote the ith outcome. These equations can be recursively solved starting with P01 = 1, P1,0 = 0. 42. (a) 1 = ∫∫ ∫∫ f (x, y)dydx = ∫ ∫ c dydx = cA(R) (x , y) ∈R where A(R) is the area of the region R. Since 0 ≤ X ≤ c, it follows that X2 ≤ cX. Let Li denote the life of component i. 1 = E[eθX]
≥ eθE[X] by Jensen's inequality. For i ≠ j ≠ k, P(Ai,jAj,k) = 365/(365)3 = 1/(365)2. Let V be the event that the letter is a vowel. (a) For a > 0 FZ(a) = P{X ≤ aY} ∞a/ y = ∫ ∫f X (x) fY (y)dxdy 0 0 ∞ = ∫F X (ay) fY (y)dy 0 ∞ fZ(a) = ∫f X (ay) yfY (y)dy 0 (b) FZ(a) = P{XY < a} ∞a/ y = ∫ ∫f X (x) fY (y)dxdy 0 0 ∞ = ∫F X (a / y) fY (y) dy 0 ∞ fZ(a) = ∫f X
(a / y) 0 1 fY (y)dy y If X is exponential with rate λ and Y is exponential with rate μ then (a) and (b) reduce to λ ∫ (a) FZ(a) = λe − λay yμe − μy dy 0 ∞ ∫ (b) FZ(a) = λe − λa / y 0 6. P{ith⎜all heads} = (i / k) n k ∑(j / k) . Letting Z = (X − 12)/2 then Z is a standard normal. Solutions Manual A First Course in PROBABILITY Seventh Edition Sheldon Ross
Prentice Hall, Upper Saddle River NJ 07458 Table of Contents Chapter 1 ....1 Chapter 2 ....10 Chapter 3 ....20 Chapter 4 ....46 Chapter 5
....64 Chapter 6 ....77 Chapter 7 ....98 Chapter 8 ....133 Chapter 9 ....139 Chapter 10
....141 Chapter 1 Problems 1. Hence, the desired probability is (n + 1/2n 23. For any given arrangement of k successes and n − k failures: P{arrangement⎜total of k successes} = 12. Chapter 2 17 Theoretical Exercises i =1 5. ⎛ 32 ⎞⎛ 52 ⎞⎢ ⎥⎢ ⎥⎝ 13 ⎠⎝ 13 ⎠35. Hence. ∞ ∑ P{N (x) ≥ n + 1} = n=0 ∞ ∑x n / n!
= e x n=0 (a) Number the red balls and the blue balls and let Xi equal 1 if the ith red ball is selected and let it by 0 otherwise. (a) P{X1 = 1⎜X2 = 1} = 4/12 = 1 − P{X1 = 0⎜X2 = 1} (b) P{X1 = 1⎜X2 = 0} = 5/12 = 1 − P{X1 = 0⎜X2 = 0} 36. Let Ai be the event that couple i sit next to each other. e−5 + 5e−5 + 78 52 − 5 53 − 5 e + e 2! 3! Chapter 6
14. 99 10 10 99 109 42. (a) Condition on whether the non-albino child is a carrier. ⎛ 26 ⎞(a)⎢ ⎥⎝ 38 ⎠5 3 ⎛ 26 ⎞12 (b)⎢ ⎥⎝ 38 ⎠38 72. Also, let Si be the event that the family chosen in method a is of size i. ⎛ 20 ⎞⎢ ⎥⎝2⎠14. 1. Now Pa,b = P{last is white⎜first a are white} + P{last is white⎜first b are black} 1 ⎛a + b⎞⎢ ⎥⎝ a ⎠1 ⎛b + a⎞⎢ ⎥⎝ b ⎠+ P{last is
white⎜neither first a are white nor first b are black} ⎤⎡ 1 1 ⎥a !b ! a !b! a !b! ⎤1 1⎡ ⎥=⎥1 − = − +⎥1 − − ⎥2 (a + b) ! 2⎣ (a + b) ! (a + b)!⎦⎦ ⎛ a + b⎞⎛b + a⎞⎢ ⎥⎢ ⎥⎝ a ⎠⎝ b ⎠⎦⎦⎣ where the induction hypothesis was used to obtain the final conditional probability above. P{X = 0} = P{1 loses to 2} = 1/2 P{X = 1} = P{of 1, 2, 3: 3 has largest, then 1,
then 2} = (1/3)(1/2) = 1/6 P{X = 2} = P{of 1, 2, 3, 4: 4 has largest and 1 has next largest} = (1/4)(1/3) = 1/12 P{X = 3} = P{of 1, 2, 3, 4, 5: 5 has largest then 1} = (1/5)(1/4) = 1/20 P{X = 4} = P{1 has largest} = 1/5 Chapter 4 47 15. Chapter 1 3 31. ∫ s 0 f (x)dx is the cumulative distribution of demand. E[X1X2⎜Y = y] = E[X1⎜Y = y]E[X2⎜Y = y] =
y2 Therefore, E[X1X2⎜Y] = Y2. 66 (a) (b) (c) (d) (e) ∫ ∫ Φ(.8333) = .7977 2Φ(1) − 1 = .6827 1 − Φ(.3333) = .3695 Φ(1.6667) = .9522 1 − Φ(1) = .1587 Chapter 5 16. Let PA be the probability that A wins when A rolls first, and let PB be the probability that B wins when B rolls first. ⎛11⎞Hence, answer is⎢ ⎥= 165 ⎝3⎠⎛7⎞(b) here it is the number of
positive solutions—hence answer is⎢ ⎥= 35 ⎝3⎠32. a dx dx A ⎛ a2 ⎞(a) E [X − a] = (x − a) + (a − x) = −⎢ a − ⎥A 0 A 2 ⎝ A⎠a ∫ ∫ d (da)= 2a −1 = 0 ⇒ a = A/ 2 A a ∞ 0 a (b) E [X − a] = (a − x)λe − λx dx + (x − a)λe − λx dx ∫ ∫ = a(1 − e − λa) + ae − λa + e − λa λ − 1 λ + ae − λa + e − λa λ − ae− λa Differentiation yields that the minimum is
attained at a where e − λa = 1 / 2 or a = log 2/λ (c) Minimizing a = median of F 32. (b) same answer as in (a). (a) 1 − e−1/2 (b) 1 −1 / 2 e 2 (c) 1 − e−1/2 = 60. Let X = 1 if the coin toss lands heads, and let it equal 0 otherwise. Consider the final round of the duel. Similarly, 1/3 = P(B⎜C) > P(B) > P(B⎜C c) However, P(AB⎜C) = 0 < P(AB⎜C c). P{F(X ≤
x} = P{X ≤ F−1(x)} = F(F−1(x)) =x 29. (c) P{X 2 + Y 2 ≤ 1} = 16. The example in Section 3 of random variables uncorrelated but not independent provides a counterexample to the converse. Part (a) remains the same. Hence, E[Xn] = E[E[Xn⎜Xn−1]] = E[μXn−1] = μE[Xn−1] so, E[Xn] = μE[Xn−1] = μ2E[Xn−2] ... = μnE[X0] = μn (c) Use the above
representation to obtain E[Xn⎜Xn−1] = μXn−1, Var(Xn⎜Xn−1) = σ2Xn−1 Hence, using the conditional Variance Formula, Var(Xn) = μ2 Var(Xn−1) + σ2μn−1 (d) π = P{dies out} = ∑ P{dies out X i = j} p j j = ∑π j p j , since each of the j members of the first generation can be thought of as j starting their own (independent) branching process. E[g(X,
Y)] = ∫ P{g (X ,Y) > a}da 0 ∞ = ∫ 0 ∫∫ g (x, y) f (x, y)dydxda = x , y: g (x, y) > a 0 = 4. (a) Let d = D/L. No 22. 14 14 36. Let X and Y denote, respectively, the number of males and females in the sample that never eat breakfast. 6! = 720 2 · 3! · 3! = 72 4!3! = 144 6 · 3 · 2 · 2 · 1 · 1 = 72 (12)! = 27,720 6!4! (a) (b) (c) (d) Chapter 1 8! = 40,320 2 · 7!
= 10,080 5!4! = 2,880 4!24 = 384 1 11. Let Xi denote sales in week i. By the generalized principle there are thus 7 · 7 · 7 · 7 = 2401 kittens 7. Now ⎝ i =1 ⎠Var(X1) = E[X 12] − (7 / 2) 2 = [1 + 4 + 9 + 16 + 25 + 36]/6 − 49/4 = 35/12 ∑ ⎛ 10 ⎞and so Var⎢ X i ⎥= 350/12.

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